

DIRECTIONS
FOR THE USE OF
HADLEY'S QUADRANT, &c.



NOTICE TO THE PUBLIC

DIRECTORY

FOR THE YEAR 1881



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DIRECTIONS
FOR THE USE OF
HADLEY'S QUADRANT,
WITH
REMARKS ON THE CONSTRUCTION
OF THAT INSTRUMENT.

By the Reverend Mr. LUDLAM.

LONDON,

Printed by R. HETT,

And Sold by T. CADELL, in the Strand.

M.DCC.LXXI.

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HADLEY

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P R E F A C E.

THE instrument here treated of is so extensive in its use, and so easy to manage, that one cannot but wonder it is not yet become more common. The seamen indeed have at last adopted it, and, dropping their old *Davis's* quadrants, use this only for finding their Latitude; so that it

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it has now got the name of the *Sea-quadrant*. The application of this instrument to the taking distances of the moon from the sun and stars, in order to find the Longitude at sea, begins likewise to be understood and practised; a business in which no other quadrant can be of the least service. But besides seamen, who are indeed obliged to observe the heavens, there are many other persons who would amuse themselves with the practical parts of Astronomy, were the necessary instruments neither expensive nor troublesome to use. This instrument is cheap, portable, applicable to many astronomical purposes, and of all others is the easiest to manage.

P R E F A C E. vii

nage. It is indeed its peculiar excellence that it requires no steadiness of the hand to be acquired only by long practice; no fixed basis as most other astronomical instruments do. Besides astronomical purposes, it makes an useful *Theodolite* for the surveying and *mapping* of counties, and far excels any other instrument for taking *off-sets* in the modern way of plotting and measuring Land. Nothing seems to have hindered this instrument from coming into general use, but the want of proper directions, not only how to use it, but also to examine the goodness and rectify the position of the glasses; without which no observations made with it can be

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of any value. The makers themselves are but little acquainted with the method of adjusting the glasses for the back observation: But how well soever a maker may adjust the glasses at first, they will never keep their position, but must be examined and rectified from time to time; so that directions on this head are absolutely necessary. Nothing is here directed to be done, but what has been actually executed: There will always be some doubt that what is proposed upon Theory only, may not be feasible when it comes to be tried.

The remarks are sometimes more proper for those that make than those that use the instrument; possibly

P R E F A C E. ix

sibly they may be of service to both. All mathematical demonstrations are omitted, as belonging to another place; what is here delivered not being designed for speculation, but for practice.

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DIRECTIONS
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HADLEY'S QUADRANT, &c.

A short history of the invention of the reflecting instrument for taking angles at sea, commonly called HADLEY'S QUADRANT, and by foreigners the ENGLISH OCTANT.

1. The first account of any instrument of this kind is in a paper given in to the Royal Society by *John Hadley Esq.* May 1731. * The author describes two reflecting octants; the latter of these has three speculums, by means of which any angle under 180 degrees may be taken, or the sun's altitude observed at sea either from that part of the horizon which is under the sun or that part which is opposite to it; nor does this octant differ essentially from those now commonly made. The author

* Philosophical Transactions, Nr. 420.

likewise explains the principles on which these instruments were constructed. In a subsequent paper dated 1732 † Mr Hadley says that an octant made of wood was produced before the society when he gave in the description, but that he had since procured another to be made of brass; and he proceeds to give a very circumstantial account of a great number of trials of them both, made on board the *Chatham* yatch, by which their great usefulness at sea was sufficiently proved; nevertheless it was at least twenty years before this excellent instrument began to come into use; so slow are even the best improvements in making their way against old prejudices.

2. Some years after this, viz. in 1742, a paper in Sir Isaac Newton's hand-writing, was found among Dr Halley's papers after the Doctor's death. This paper contains a drawing and description of an instrument not much different from the first of Hadley's; its principal properties and uses at sea are likewise there taken notice of. * It seems therefore that in reality Sir Isaac Newton was the *first* inventor of these reflecting

† Philos. Transf. Nr. 425. * Philos. Transf. Nr. 465.

quadrants, though this was not known till 1742, except perhaps to Dr. Halley, who seems to have taken no notice of this paper of Newton's, when Hadley's octant was publicly shown at the Royal Society. Mr Hadley's great abilities and particular skill in optics (of which there are many proofs in the Transactions) leave no room to doubt but that he likewise was an original inventor; and accordingly the instrument has always born his name.

An account of the several sorts of reflecting instruments for taking angles.

3. There are several sorts of these instruments for taking angles by reflection now commonly made. Large ones of brass are apt to bend across their plane unless made with perpendicular bars and then they are very heavy. * The common sort are of mahogany: their divisions are cut upon ivory, which if it does not shrink may do very well, as it shows the lines very plain and will not rust with the sea air. The best size and sort for general use seem to be those

* Perpendicular bars are those whose plane is perpendicular to the plane of the quadrant, and are fastened on the backside of it.

of fifteen inches radius made of mahogany with a brass plate on the limb for the divisions.

4. Some of these instruments are octants, others sextants. The octants when furnished for the back observation will take any angle under 180 degrees, and answer all the purposes of a semicircle. The sextants are never furnished for the back observation and therefore take in no more than 120 degrees and that with difficulty, as the extreme reflections are very oblique and consequently the field small. They are besides more cumbersome and heavier than the octants. It would be well therefore if those who use this instrument would accustom themselves to take back as well as fore observations, and learn to examine and adjust their instrument for both purposes. * Very small instruments (of seven or eight inches radius) cannot well be fitted for the

* If we may guess by the drawing, (Phil. Trans. Nr. 420) in the first octants the little speculum for the back observation was placed so near the index glass, that an altitude of the sun (unless very high) could not be taken, the head of the observer being in the way. Seamen finding the back observation frequently impracticable in the first instruments, would naturally be prejudiced against it in all others; which I believe is the chief reason why the back observation has been so much neglected.

back observation, and are therefore commonly sextants. These are always made of brass, and in so small a size are strong enough without perpendicular bars. A brass octant of twelve inches radius made thick will do without perpendicular bars, the narrowness of the shape making it stronger across the plane than a sextant. An octant of this size has just room enough to admit the glasses for the back observation.

Description of a reflecting Octant.

5. Figure 1 represents the octant furnished both for the fore and for the back observation. *AB* and *AB* are the two sides, *DF* is the arch or limb, *MN* is the moveable arm called sometimes the *Alidade* but commonly the index; this turns on the center *C* and carries the index glass or great mirror *I*. The end *N* of the index shows the degrees and minutes upon the limb as will be explained hereafter. *H 1* is the horizon glass or little mirror for the fore observation. *H 2* is the horizon glass or little mirror for the back observation. *O 1* is the sight vane for the fore observation. *O 2* the sight vane for the back observation. *L 1* is the place of the red glasses for the

6 DESCRIPTION OF AN OCTANT.

fore observation; these are removed to *L 2* for the back observation.

6. The index glass *I* is put into a brass frame and is drawn tight against an upright plane of brass by two screws *e* and *f* behind the frame. To the bottom of this upright plane is joined another brass plane at right angles to it, by which the whole brass work carrying the index glass is screwed down to the index.

7. This brass work is screwed to the index by three screws behind the glass at *C* fig. 1. and again represented in fig. 2. Two of these *a* and *b* draw the work down to the index, the third *c* pushes against the other two. These screws serve not only to fasten the brass work but also to incline the index glass backwards or forwards in order to adjust it.

8. Sometimes this brass work is screwed fast to a circular plate of brass which rests upon two points placed underneath the ends of the index glass. The screws *c* and *d* (fig. 2) fasten this circular plate down upon those points, and by drawing against each other, incline that plate and the glass upon it backwards and forwards in order to adjust it as before.

9. The

DESCRIPTION OF AN OCTANT. 7

9. The fore horizon glass is likewise put into a brass frame and kept tight by two small screws behind the frame, like the index glass.

10. The brass work containing the horizon glass is screwed fast to a circular plate of brass by two screws *bb* fig. 3 behind the glass. This circular plate rests upon two points in another circular plate underneath the former; these points are placed under the ends of the horizon glass.

11. There are two adjusting screws *ss*, one before and one behind the glass, which draw the circular plate down upon the points. These adjusting screws by drawing against each other, serve to incline the glass backwards and forwards in order to adjust it.

12. Besides the adjusting screws *ss*, the horizon glass has likewise an adjusting lever by which it may be turned circularly round an axis perpendicular to the plane of the octant. This lever is usually on the back-side of the octant and has a contrivance to turn it slowly, and a button-screw to fix it. In some instruments this glass is turned about by an endless screw.

13. The lower half only of the fore horizon glass is silvered over, the upper half

8 DESCRIPTION OF AN OCTANT.

is left clear, through which, in making observations, one of the objects is seen directly, while the other is seen by reflection from the lower half of the glass. *

14. The back horizon glass is fixed in a frame in the same manner and has the same adjusting screws and adjusting lever as the fore horizon glass. It differs only in this, that it is silvered both at the top and bottom, and has a transparent slit across the middle, through which one of the objects is seen directly while the other is seen by reflection. †

15. The fore sight vane is made in various ways. Generally it is only a flat plate of

* If the octant was to be used for land objects only, it would be better if the upper half of the horizon glass was cut off. For though land objects may be seen through it, yet they appear much darker. The great use of this part of the horizon glass is in observations of the sun, whose image is bright enough to be seen by reflection from this part though not silvered: and thus that reflected image can be made to coincide exactly with other objects seen directly through the very same point of the glass.

† I suppose the reason of thus leaving only a slit transparent, to be as follows. The eye (when a plain hole is used for the sight) being very near the glass, a narrow slit is sufficient to give a large field for objects seen by direct vision: the remaining part of the glass both above and below is silvered, that the objects to be seen by reflection may be more easily found. This construction however is inconvenient, especially for land objects when either a plain tube or telescope is used. This horizon glass should undoubtedly be silvered just in the same manner as the other.

brass

DESCRIPTION OF AN OCTANT. 9

brass with two holes in it. One hole is exactly over against the separation between the silvered and transparent part of the horizon glass; the other hole is above the former and opposite to the middle of the transparent part of the horizon glass. The lower hole is to be used in common cases; the upper hole when the object is so bright as to be seen by reflection from the unsilvered part of the glass. There are little shutters to cover the hole not in use.

16. Instead of the vane with holes, a plain tube is frequently used to direct the sight, and sometimes a telescope. When a plain tube is used, its height should be changeable so that its axis may be made to point either against the separation between the silvered and transparent part of the horizon glass, or else against the middle of the transparent part, which is necessary in taking the altitude of the sun at sea. The telescope is always made to rise and fall above and below the separation of the silvered and transparent parts of the horizon glass, so as to take in more or less of the one or the other. By this means either object (that seen directly or that by reflection) may be made to appear through the telescope more

or

80 DESCRIPTION OF AN OCTANT.

or less bright as circumstances may require. This telescope is likewise furnished with smoaked glasses in a brass frame which slides across the eye hole. This brass frame contains two glasses whose inward faces are smoaked with the flame of a candle; one end lightly, and then gradually darker and darker to the other end. A frame of brass is interposed between the smoaked glasses to prevent their touching each other so as to rub off the soot. These smoaked glasses are to be used in viewing the sun when both its direct and reflected images are brought into the telescope together.

Some instruments are furnished with all three kinds of sights, plain holes, a tube, and a telescope, to be put on and used in each others place.

17. The back vane does not differ in its construction from the fore vane. A telescope is seldom applied in the back observation, though provision might easily be made for applying the same telescope in the back observation that is used in the fore observation; and it would be an improvement of the instrument to do so. *

18. The

* It may be remarked that in common octants the sight
vanes

DESCRIPTION OF AN OCTANT. 11

18. The two objects which coincide are seen, the one by a ray passing through, the other by a ray reflected from the same point of the horizon glass to the eye; this ray is called the *Visual ray*. When this ray is considered merely as a line drawn from the middle of the horizon glass to the eye-hole of the sight vane, it is then called the *Axis of Vision*. The axis of the telescope or tube used to direct the sight, coincides with, and is also considered as the *Axis of Vision*.

19. The red glasses (or as some call them, dark glasses,) are round glasses put into square frames of brass. These glasses are not made fast in their frames but may be turned about to adjust as will be shown hereafter. Their use is to make the object seen by reflection appear of a dark red colour, when it would otherwise be too bright. One of these glasses is of a lighter, the other of a darker colour; the former is to be used in taking distances of the moon from the stars in order

vanes are made with holes only, and those very small. Being chiefly designed for taking the sun's altitude at sea, the brightness of the object makes it easy to find the reflected image in the horizon glass, and small holes are sufficient to show it: but for land objects a short tube is very necessary to take off the light of the sky, which entering sidewise through a plain hole almost effaces the light coming from the object.

to

12 DESCRIPTION OF AN OCTANT.

to find the longitude at sea, the latter in observations of the sun. If the sun is very bright both red glasses may be used together. These glasses turn out of the way in other observations.

20. The limb DF is divided into degrees and half degrees, every ten degrees of which is numbered, beginning from 0 at E and going towards the left hand to 94 or 95 degrees. This part of the limb is called the *quadrantal arch* or *internal arch*. The part of the limb EF lying the contrary way from 0 is likewise divided into degrees and half degrees and is numbered from 0 at E the contrary way towards the right hand. This part EF is called the *arch of excess* or *external arch*. The odd minutes are shown by a scale of divisions VV on the index called sometimes a *Nonius* and sometimes a *Vernier*. *

21. The middle line of this scale of divisions or Vernier, is distinguished by being longer than the rest; against this line is set 0.

* This invention has been commonly attributed to *Petrus Nonius*, (or *Pédro Nuñez*) and it has accordingly born his name; but it was in reality the invention of *Pierre Vernier* a Frenchman in 1631; and of late both French and English writers have called it by his name.

DESCRIPTION OF AN OCTANT 13

The other divisions are numbered, every five, from 0 to 15 on the right hand, and then beginning from the extreme division on the left hand from 15 to 0, thus:

15 20 25 0 5 10 15.

22. The place of the middle line of the Vernier among the divisions on the limb, shows the intire degrees and odd half degree (if any) subtended by the objects observed. The odd minutes (always under 30) are shown by the Vernier in manner following. Look among the strokes on the Vernier for one that stands directly opposite to some stroke among the divisions on the limb; that stroke numbered among the divisions of the Vernier shows the number of minutes to be added to the intire degrees and half degree (if any) shown as aforesaid on the limb.

23. If the angle subtended by the objects is taken upon the arch of excess, so that the middle line of the Vernier falls between *E* and *F*, then in finding the odd minutes by the Vernier, the numbers on the scale are to be read in a contrary order, just as if they had been figured thus,

15 10 5 0 25 20 15:

or

24. DESCRIPTION OF AN OCTANT.

or which is the same thing, you may first read off the odd minutes in the usual way, and then their complement to 30 will be the real number of minutes to be added to the degrees and half degree shown on the arch of excess. See also par. 27.

24. What has been said relates intirely to the fore observation; in the back observation, the angle subtended by the two objects is the *supplement to 180 degrees* of that shown on the limb, and estimated by the figures engraved thereon, as aforesaid.

25. In taking the sun's altitude at sea by the back observation, the two objects whose angle is determined are the sun, and that part of the horizon which is opposite to the sun: but the supplement of this angle, is that which is contained between the sun and the part of the horizon under the sun; therefore in such back observations at sea, the numbers shown on the limb immediately give the sun's altitude, just as in the fore observation.——In other cases of the back observation the angle between the objects is had by subtracting the degrees and minutes shown (and estimated by the figures)

DESCRIPTION OF AN OCTANT. 15

figures) on the limb, from 180 degrees as was said before. *

26. The index has a setting screw on the backside to fix it; nevertheless after the index is thus set, it may yet be moved slowly backwards or forwards by the regulating screw *R*, fig. 1, by which means the two objects may be nicely brought into contact. Some instruments have only a setting screw and no regulating screw, but this is a defect.

27. In some instruments the Vernier is numbered in a manner different from that we have described. The figure 0 stands against the extreme division at one end, commonly on the right hand, and the numbers run regularly on to the other end, thus:

30 25 20 15 10 5 0;

but whenever the angle is taken on the arch of excess, these numbers must be read in a contrary order, thus:

* To make the instrument have the perfect semblance of a semicircle, the numbers on the limb should be returned back again from *D* to *E*, that is from 90 to 180 degrees. There should likewise be another set of numbers on the Vernier in a contrary order: but as so many sets of figures might create confusion, it is better to have one set of figures only as usual; to read off the degrees and minutes in the common way, and take their supplement to 180 degrees in the back observation.

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0 5 10 15 20 25 30: as was before directed. *

28. The degrees on the limb are likewise differently subdivided in different instruments; this will occasion a difference in the scale of the Vernier. When the degrees are subdivided into three parts, each part contains 20 minutes, and the numbers on the Vernier run only to 20 not to 30; thus; 10 15 0 5 10; or thus; 20 15 10 5 0. All the foregoing rules hold in this case. For instance, if the angle be taken on the arch of excess those figures must be read thus;

10 5 0 15 10;
or in the latter case thus;

0 5 10 15 20.
Otherwise the numbers may be read just as

* The 0 may be made to stand at either end, and the numbers to read either in the same order with the figures on the limb, or in the contrary order. The length of the intire scale of the Vernier must be equal to as many of the least parts into which the limb is *actually* divided, as are contained in the Vernier and *one more* in the latter case, but *one less* in the former. Thus where the limb is actually divided into half degrees, and the Vernier of course has 30 parts to distinguish each half degree into minutes; the length of those 30 parts of the Vernier, must be equal to 31 half degrees on the limb when the figures on the Vernier are to be in an order *contrary* to those on the limb; and to 29 half degrees when the figures on the Vernier are to be in the *same* order with those on the limb.

DESCRIPTION OF AN OCTANT. 17

they are engraved on the Vernier, and their complement to 20 taken for the true number of minutes to be added to the odd 20, or the odd 40 shown on the limb.

29. In some instruments the index is so made that the scale of the Vernier is in the same plane with the scale of degrees on the limb; in others the scale of the Vernier comes down to the limb with a slope or feathered edge. The coincidence of the strokes may be more exactly seen when both scales are in one plane, provided the limb be wrought so true that the edge of the Vernier makes a perfect joint with the limb in every part: But as this is scarce possible, the method of making the Vernier with a feathered edge seems preferable.

In some instruments the index moves underneath the plane, that is on the back side. This is very inconvenient; for the instrument can never be laid down without danger of disturbing the index; indeed good instruments are seldom made in this form.

30. It is usual to varnish all the brass work of a bright gold colour; but it would be better if the frames that surround the glasses, and the parts on which they stand

C

were

18 DESCRIPTION OF AN OCTANT.

were varnished with a dead black. The glare of the brass-work in the former case gives a false light which enters the eye along with the light from the objects and in a manner overcomes it.

Of a Support for the Octant.

31. Although that degree of steadiness which is necessary in all other instruments is not required in this, yet a proper stand to support the weight of it is very convenient. One of this sort is described in the *Astronomical observations made at Cambridge &c.* * That stand is particularly designed for taking distances of the moon from the sun or stars, or the distances of stars from each other. A far more simple apparatus will serve when the octant is to be used only to take angles between land objects in making surveys. The part marked *QRST* in plate first of that work (and here in figure 4) is alone sufficient for this purpose. The axis *QQ* may be put thro' the socket of the three-legged staff made to support a common *Theodolite* in the very

* Printed for T. Cadell, London.

A STAND FOR THE OCTANT. 19

place of the long screw that screws the Theodolite down upon the socket. This apparatus may be taken away and the Theodolite put upon the socket occasionally.

32. Fig. 4. is the profile of an apparatus of this sort made for a twelve inch octant. The axis VV on which the octant swings is one intire piece of brass, and is screwed on the back side of the octant. It should be so placed as to pass through the center of gravity of the whole instrument and be parallel to the axis of vision belonging to the fore horizon glass.

Care should be taken that the supports or brass heads SS do not lie in the direction of the axis of vision of the back horizon glass. For then these heads will intercept the direct view through the transparent part of that glass when the face of the octant is downwards: * for the octant may be placed
on

* It is to avoid this that the axis VV is made parallel to the axis of vision of the fore, and not of the back horizon glass, nor in an intermediate direction. Another reason is, that notwithstanding the axis VV is not parallel to the axis of vision of the back horizon glass, yet if the octant be made to vibrate about the axis VV the reflected image of any object seen in the back horizon glass will describe an arch, the convex side of which is turned towards those objects whose angular distance from the given object exceeds that of its image. The advantage of this will be better understood hereafter.

on such a support with its face either upwards or downwards, and thus the faintest of the two objects be always seen directly and the brightest by reflection.

A small addition to this work will make it a useful stand for the octant when applied to the taking altitudes of the sun or stars by reflection from the surface of a fluid.

Of the apparatus for taking altitudes by reflection from the surface of a fluid.

33. It is necessary to screen the surface of the fluid from the action of the wind by two planes of glass set over the fluid so as to be nearly perpendicular to the rays transmitted through them. These planes are each three inches broad and four inches long, and are pinned in mahogany frames. The sides and bottom of the frames are half an inch wide, the top one inch wide. The frames are joined together by hinges screwed to the top. To the top of each frame is likewise screwed a brass arch, by which they can be set

An octant of less than 12 inches radius, cannot well be mounted in this manner, because its center of gravity will fall too high, and the axis *VV* pass too near the sight vanes.

to

to any inclination between 180 and 50 degrees. The figures on the arch show half the inclination of the planes; therefore the planes must be set so that the index may point on the arch to the expected altitude. The perspective view of these planes is given in Fig. 5. and the plan of the brass arches in Fig. 6.* These brass arches slip beside each other and have each a long circular slit, through which passes a setting screw. The profile of this screw is seen in Fig. 7. On the inside of the head of the screw is a steady pin which goes through the slit of one of the arches and keeps the head from turning round. A small part *ab* of each arch returns at right angles to the plane of the other part, by which it is screwed to the mahogany frame.

34. Fig. 8 is the perspective view of the reflecting box, and Fig. 9 is the profile to one fourth of the real size; the sections of the planes are expressed by dotted lines but without their brass arches; these sections

* Figures 2 3 4 6 7 9 are profiles or sections, laid down from the original by scale and compass. Fig. 1 5 8 10 11 are perspective views: their principal lines were marked out mechanically, with an instrument by which you may draw the perspective view of any object with ease, expedition and accuracy.

are marked by *LE* and *LF*, in fig. 9. The reflecting box is dove-tailed together at the corners *AB* and *CD*, but open every where else and without a bottom. It is four inches wide, that is just enough to admit the planes which rest upon the thin boards *AE* and *CF*. These boards slide into grooves made in the sides of the box. The brass pin *MN* fig. 8 keeps the top of the box from warping or flying out. The water trough *p q r s* is made of pewter; four inches long, three inches wide and two inches deep, and is blacked within. It has a cover which skreens about a quarter of an inch of the water next the sides of the trough all round, to hide the irregular reflections made by air-bubbles which adhere to the sides. There is another trough of wood of the same dimensions to be used in the place of the former: this has a false bottom within half an inch of the top to hold quicksilver. When the altitude of the star is very great, the trough may then be covered with one of the planes laid flat upon it; the mahogany frame being made to fit the top of the trough for that purpose.

35. The two surfaces of each glass plane should be perfectly parallel, though a small variation is of no consequence; partly because the rays pass almost perpendicularly through them, and partly because by turning the planes, their refractions will have contrary effects and the errors will balance each other, as will be shown hereafter. The parallelism of these glass planes may be examined by the methods described in the following paragraphs.

THE METHOD OF EXAMINING THE GLASSES BELONGING TO THE OCTANT.

All the glasses belonging to this instrument should have their two surfaces perfect planes and exactly parallel to each other.

36. To examine whether the two surfaces of any of the reflecting glasses are parallel, observe the image of some distant object reflected very obliquely from the glass; * if that image appears single it is a sign that the two surfaces are parallel. † On the contrary, if the edges of the object thus seen by reflection appear to be separated so as to make two images, the surfaces of the reflecting glass are then inclined to

* By distant objects we shall for the future understand *first* and *principally* any object in the heavens, the sun, moon, or stars. *Secondly* the sea horizon. *Thirdly* land objects not less distant than a quarter of a mile; but the more distant these are, the better.

† For the distance between the two images reflected by the two surfaces (when parallel) is twice the thickness of the glass; neglecting the refractions between the surfaces which lessen the distance between those images. If the object be so far off that this distance subtends an angle less than a minute, and is consequently invisible, the two images will then appear as one.

each

each other. * This examination will be more perfect if the reflected image be viewed through a telescope magnifying eight or ten times. If the image then appears single, its parts sharp and well defined; it is a sign that the reflecting glass is a good one indeed,

37. It may be proper to take notice of the reflected image of the sun or moon when the index stands at 90 degrees, the reflection from the index glass being then the most oblique possible. If the sun's image then appears single, it is a sign the glasses are good.

38. But the surfaces of the reflecting glasses should not only be parallel but likewise perfect planes. † To examine this,

* If the surfaces either of the index glass or the horizon glass should not be exactly parallel, but, when produced, intersect each other, yet if these glasses are so placed that this intersection is parallel to the plane of the instrument it will not occasion any error in taking angles. For though there will be two images formed, yet these two images will be one below the other with respect to the plane of the octant, and therefore each at the same angular distance from the object itself.

† The reflecting glasses are often distorted by the screws which draw the brass frame tight against the glass and by that means press the glass against the upright plane, See par. 6 and 9. Sometimes the face of this plane, sometimes the glasses themselves *cross-wind*. In truth both this brass work and the glasses themselves are generally made much too thin to be capable of being wrought true.

look

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look out for two distant objects: move the index till the reflected image of one object coincides with the other object seen directly; making them unite just at the separation between the silvered and transparent part of the horizon glass. Turn now the octant about *in its own plane*, so as to make the united images move along the line of separation in the horizon glass. If the images continue united, then the reflecting planes are flat; but if the image of one object either passes beyond or recedes from the other in the direction of the line of separation in the horizon glass, (that is in the direction of the plane of the octant) the glasses are faulty. Any separation of the images in a direction perpendicular to the plane of the octant is of no consequence.

39. In this examination care must be taken to hold the octant exactly in a plane passing through the eye of the observer and the two objects, as will be more particularly insisted upon hereafter. If either of the objects is bright enough to be seen by reflection from the transparent part of the horizon glass, its image may be brought to the upper part of the horizon glass by holding
ing

ing the octant out of the proper plane. The other object may likewise be seen at the same time through the upper part of the horizon glass: but though they were united before in the middle of the glass, they will not be united now in this upper part, not owing to any fault in the glasses but to the wrong position in which the octant is held. It requires experience in the management of the instrument to make this examination properly, so as not to mistake that for a fault in the glasses which is only a fault in the manner of holding the octant. See par. 108.

Of the Red Glasses.

40. The examination of these glasses must be made when the sun is on or near the meridian. Hold the octant nearly in a vertical plane and direct the sight to some object in the horizon under the sun. Turn down the red glass and move the index till the reflected image of the sun is in contact with the object seen directly. Fix now the index and turn the red glass round in its brass cell: if this neither raises nor depresses the sun's image, but it continues in contact with

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with the object below as before, then the surfaces of the red glass are parallel. If turning this glass should raise or depress the sun's image, put the glass into that position which leaves the image in the middle between the highest and lowest points to which it shifts it, so that the refraction may remove the image of the sun sidewise along the horizon in a direction perpendicular to the plane of the octant. In this position of the red glass, its refraction will not make the instrument false. *

41. This examination is here directed to be made when the sun is on the meridian, because his distance from the object beneath does not vary sensibly at that time for several minutes: but it may be made at any other time, provided the plane of the octant be held in the plane of the hour circle passing through the sun, and the object to be seen directly be placed in the intersection of that circle with the horizon.

* If the two surfaces of the red glass be not exactly parallel, the rays of light (in passing between the index and horizon glasses) will not be parallel before and after refraction at the red glass. But if the intersection of the surfaces of that glass, when produced, be parallel to the plane of the octant, this want of parallelism in the rays will not occasion any error in the angle observed; for the refraction of the glass will then be wholly perpendicular to the plane of the instrument.

42. The

42. The same trial of the red glaffes, especially the faint one, may be made by the moon, by bringing her reflected image into contact with some fixed star, lying nearly in a great circle drawn through the moon, at right angles to her course in the heavens. For the distance from such a star will not be sensibly varied for many minutes by the proper motion of the moon in her orbit.

THE METHOD OF ADJUSTING
THE GLASSES AND OTHER
PARTS OF THE OCTANT.

43. Before you begin to adjust the position of the glasses, you must first examine whether they are all fast in their frames, if not you must fix them by the screws behind. See par. 6 and 9. But take care not to force these screws so as to bend or break the glasses. See likewise that the brass work in general is every where firm and fast, that the several adjusting screws which act against each other are just tight and no more.

To adjust the Index Glass.

44. We shall begin with the index glass. Now to adjust the index glass is to set its plane perpendicular to the plane of the instrument. This is performed by means of two adjusting tools made of brass and represented in perspective, Fig. 10 and 11. One of these tools, Fig. 10, consists of two planes joined at right angles to each other,
one

one of which we shall call the horizontal plane, the other the vertical plane, from their positions when in use. In the vertical plane is a small hole for the eye to look through. The other tool, Fig. 11, consists of an horizontal plane like the former, at each end of which, and close to one side, is an upright pillar: between these two pillars is stretched horizontally a silver wire or white horse-hair, exactly at the same height with the sight-hole of the other tool; so that when the tools are set close together and their vertical planes touch, the wire in one bisects the hole in the other.

45. To adjust the index glass then, first take the fore-sight vane clear off the octant, so that the index may pass over the place of that vane. Set the former of these tools, Fig. 10, on the arch or limb, set the latter, Fig. 11, on the sides of the octant as near the index glass as may be, but not upon the index itself. Turn now the index clear off the limb, so that by looking through the hole in the former tool, Fig. 10, you may see that hole by reflection from the index glass; at the same time you are to see the wire of the other tool, Fig. 11, directly.

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If the reflected image of the hole you look through is bisected by the wire, then the index-glass is upright. If the hole appears above the wire, then the index-glass inclines forward; contrariwise, if the hole appears below the wire. In this case the adjusting screws (par. 7 and 8) must be the one released the other tightened, till the hole seen by reflection is bisected by the wire.

46. It may be proper to repeat this examination, setting the tool, Fig. 10, that has the eye-hole, on a different part of the limb. If the plane of the instrument is perfectly flat, the reflected hole will appear bisected by the wire wherever that tool is placed: but great precision in this particular must not be expected, and indeed need not be required.

47. In this adjustment take care always to release one screw before you set up the other that draws (or otherwise pushes) against it. Likewise be sure to leave all the adjusting screws tight, by screwing them up if slack, so that at last they may draw with a moderate force against each other. *

To

* These adjusting screws ought to move very freely, other-

To adjust the fore horizon glass.

48. Two adjustments belong to the fore horizon glass, one to set it perpendicular to the plane of the octant, the other to make it agree with the divisions on the limb. The former of these is done thus.

The index-glass being first adjusted as before, bring 0 on the Vernier exactly against 0 on the limb, that is set the index to the beginning of the divisions: † Hold now the plane of the octant parallel to the horizon, and observe the image of any distant object in the horizon at land, or at sea the horizon itself. If the image seen by reflection is higher than the object itself seen directly, release the fore screw mentioned par. 11, and set up the back screw, and contrariwise if the image seen by reflection is lowest, till both are of an equal height, so that by shifting the index you can make both images exactly agree and appear as

otherwise it is impossible to feel how much they draw against each other. Their threads ought to be deep that the screws may not be forced out, and fine that they may not release themselves;—but they are commonly very ill made.

† Some octants have a small brass button on the edge of the side. If this be turned up, it will stop the index just at the beginning of the divisions.

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one object. In making this adjustment observe the caution before given par. 47.

49. If the sea horizon be used for this adjustment, hold the plane of the octant parallel to the horizon as before, and observe whether the two images of the horizon coincide. This will be known by remarking whether the edge of the sea, seen in the silvered part of the glass, joins in one straight line with the edge of the sea seen through the transparent part of the glass. If the former appears either higher or lower than the latter, then the horizon glass must be adjusted by the rule in the last paragraph.

50. This adjustment may be made by means of the sun, moon, or a star, in which case the octant may be held perpendicular to the horizon. If the image seen by reflection in that case appears on one side, to the right or left of the object seen directly, then the horizon-glass wants to be adjusted. In whatever plane the octant be held, if the reflected image of the star appears above the direct image *relatively to the plane of the octant*, then release the fore screw and tighten the back screw, and contrariwise.

If

If this adjustment be made by a star, slide the index backwards and forwards; observe if the reflected image in its motion passes over, that is upon the direct image. If not, the horizon-glass must be adjusted as before till it does, so that, by stopping the index in the proper place, both images may precisely coincide, and the star appear as one.

51. The other adjustment of the horizon glass is the setting it so that the index may show upon the limb the true angle between the objects. This is by far the most important of all the adjustments, and is often styled emphatically *adjusting the instrument*. This adjustment cannot always be done immediately before an observation, and if done some time before is liable to alter, so that the most experienced observers advise us not to try at an exact adjustment, but to find the error occasioned by the want of such an adjustment, and allow for it in the calculation. We shall lay down both the method of adjusting the octant perfectly, and of finding the error when this adjustment is not perfect, premising that all the other adjustments of the glasses before men-

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tioned are to be made first, and as perfect as possible.

52. To *adjust* then the fore horizon glass, turn the index till 0 on the Vernier stands exactly against 0 on the limb, and fix the index there. * Direct now the sight to the moon, a star, or some distant object, and observe whether the image seen by reflection coincides perfectly with the object itself seen directly, so as to form one appearance; if not, bring them into one by turning the adjusting lever belonging to the horizon glass, par. 12, and then fix the lever by the button-screw.

53. If the sea horizon be used for this adjustment, then hold the octant in a vertical position, and observe whether the edge of the sea, seen in the silvered part of the glass, joins in one straight line with the edge of the sea seen through the transparent part; if not, bring them into one straight line by turning the adjusting lever, then fix that lever as before and the octant is adjusted.

* Some octants have a small brass button on the edge of the side. Turn this up and the index will stop against it just in the proper place for this adjustment.

To find the Error of this adjustment, commonly called the INDEX ERROR.

54. *Method 1.* Turn the index about till the moon, a star, or some distant object, as before, appear as one by the union of the reflected image with the object itself seen directly, then the number of minutes by which 0 on the Vernier differs from 0 on the limb is the error of the adjustment, or the *index error*. If 0 on the Vernier stands between 0 and 90 on the limb, that is on the quadrantal arch (see par. 20) then this error is to be subtracted from the angle shown on the limb by the index; but if 0 on the Vernier stands on the other side of 0 on the limb, that is on the arch of excess, then the index error is to be added to the angle shown on the limb.

55. To find the index error by the sea horizon, hold the octant in a vertical position as before, and turn the index till the edge of the sea, seen in the silvered part of the horizon glass, joins exactly in one straight line with the edge of the sea seen through the transparent part, and the number of minutes by which 0 on the Vernier

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differs from 0 on the limb is the index error to be added or subtracted according to the rule just before laid down, par. 54.

In reading off the minutes shown on the arch of excess, remember the caution in par. 23.

56. *Method 2.* Take the diameter of the sun both on the quadrantal arch and arch of excess, half their difference is the index error. * This error is to be subtracted, if the diameter measured on the quadrantal arch exceeds the diameter measured on the arch of excess; if contrariwise, this error is to be added to the angle shown on the limb. In taking the sun's diameter on the arch of excess remember the rule in par. 23, relative to reading off the odd minutes by the Vernier.

It will be proper on this occasion to use the telescope. Whether that or plain sights are used, it will be necessary to apply a smoked glass to defend the eye from the rays of the sun.

The index error may be found in like manner by taking the distance of two stars, or taking the diameter of any distant land

* Half their sum is the sun's true diameter.

object,

object, both on the quadrantal arch and arch of excess.

57. To prove any of these methods, set the index so as to show the exact error found by that method; and to show it upon the quadrantal arch if that error is to be subtracted, but to show it upon the arch of excess if that error is to be added. Then fix the index, and directing the sight to some distant object, take notice whether the direct and reflected images perfectly coincide or not. If they do, the index error is truly determined; if not, it must be corrected by the methods before laid down.

58. The horizon glass may likewise be adjusted within doors by means of *two* near objects; but as in the distant ones a single object is sufficient, it will never be difficult to find one proper for this purpose abroad.

To adjust the back horizon glass.

59. The horizon glass for the back observation requires the same two adjustments as the fore horizon glass. The former of these, namely the setting it perpendicular to the plane of the instrument, is to be done in a manner like that directed for the fore hori-

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zon glass, par. 48. First take the fore-sight vane clear off the octant, so that the index may pass over the place of the vane. Hold the plane of the octant parallel to the horizon, and looking through the back vane (and through the transparent slit in the horizon glass) direct the sight to some distant object in the horizon. Turn now the index clear off the limb till you see the image of the same object by reflection from the silvered part of the horizon glass, and make this image coincide as near as possible with the object itself seen directly. If by moving the index you can make both image and object coincide perfectly, then the glass needs no adjustment of this sort; but if the reflected image stands a little above or a little below the object seen directly, then the glass wants to be adjusted. Release then one of the adjusting screws (par. 11 and 14) and set up the other till both image and object are of an equal height, so that by shifting the index you can make them both agree and appear as one single object. Observe the caution before given, par. 47.

This adjustment may likewise be made by the sea horizon, or heavenly bodies, in
a man-

a manner so perfectly resembling that described in par. 49 and 50, that we need not repeat it.

60. The other adjustment of the back horizon glass is the setting it so that the index may show upon the limb the true angle between the objects. This is equally important with the corresponding adjustment of the other horizon glass. What was observed of that in par. 51 may be applied to this. As this adjustment of the back horizon glass is very important, and yet but little known, we shall be particular in describing several methods of doing it both at land and sea; and shall likewise show how to find the index error when this adjustment is not perfect.

Methods to be practised at land.

61. *Method first.* Look out for a part of the horizon that has two distant objects upon it, on two points of the compass either exactly opposite or nearly so. Turn the index till 0 on the Vernier stands exactly against 0 on the divisions on the limb, and fix the index there. Hold now the plane of the octant level with its face upwards,
and,

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and, directing the sight to one of these objects, look in the silvered part of the horizon glass for the reflected image of the other which is then behind you. If the direct and reflected objects do not exactly coincide, change your place, moving nearly at right angles to the line joining the two objects, till they do coincide. Invert now the octant, that is turn its face downwards, but hold its plane level. Direct the sight to the same object as before, and observe if the object behind you (seen by reflection) exactly coincides with that seen directly which is before you. If the two objects coincide, now the octant is inverted, as they did before, the horizon glass needs no adjustment. If the objects do not coincide, turn the adjusting lever till you bring the object seen by reflection half way towards that seen directly. Shift now your place a little, as before, till the objects again perfectly coincide, the face of the octant being still downwards. When by shifting your place the objects are made to coincide in that position of the octant, then turn its face upwards as at first. If the objects now coincide also, the adjustment is perfect; if
not,

not, by means of the adjusting lever, bring the object seen by reflection half way towards that seen directly, and repeat the whole process as before.

62. By *distant objects*, in this particular process, are to be understood objects not more than four or five miles distant from the observer; otherwise shifting his place, as before directed, will not make them coincide. Nor should these objects be nearer than half a mile: for then any little difference in the way of holding the octant will alter the coincidence, even though the observer stands on the same spot. *

63. In all this process the observer is constantly to keep his face towards one and the same object, which is always to be seen directly, and the other object always by reflection.

64. It will be convenient always to observe with the left eye when the face of the octant is upwards, and with the right eye when the face of the octant is downwards.

* This adjustment might be better made by very distant objects without changing the place of the observer, provided the horizon was distinguished by such a variety of objects, that those parts of it which seem to coincide at each change in the position of the octant, could always be ascertained.

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The contrary may be done, but there is danger least the hair or wig of the observer should hide the object to be seen by reflection, which lies behind him.

To find the error of the adjustment or index error.

65. *Method first.* Having found two distant objects in the horizon, * nearly on opposite points of the compass, hold the octant level with its face upwards and direct the sight to one of these objects, then turn the index till the other object (which is behind you and must therefore be seen by reflection) coincides with that seen directly, and mark the degrees and minutes shown on the limb. Turn now the face of the octant downwards, and, directing the sight to the same object as before, move the index till the other object seen by reflection coincides with that seen directly, and again mark the degrees and minutes shown on the limb. Half the difference of these two arches, thus shown on the limb, is the error of the adjustment. One of these arches will be a portion of the quadrantal arch,

* The objects cannot be too far distant in this case.

and

and the other a portion of the arch of excess. In reading off this last arch observe the rule in par. 23. In every back observation I suppose the angle between the objects to be estimated by the rule in par. 24; that is, the degrees and minutes contained in that angle are the supplement (to 180 degrees) of the degrees and minutes shown on the limb, and estimated by the figures engraved thereon: But I suppose the small arch aforesaid to be read off in the usual manner, by the figures on the limb. This premised, the error of the adjustment found as before is to be added to the angle between the objects, when the portion of the quadrantal arch exceeds the portion of the arch of excess. If contrariwise, this index error is to be subtracted.

66. To prove whether the index error has been rightly found, set the index to that number of degrees and minutes to which it amounts; but so as to show it upon the quadrantal arch, if the larger of the two small arches mentioned before, par. 65, was a portion of the quadrantal arch; otherwise set the index so as to show the index error on the arch of excess, and
fix

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fix the index there. Then, by the method in par. 61, observe whether the same two distant objects, on opposite points of the compass, coincide both in the direct and inverted positions of the octant; for they ought to do so, if the index error has been rightly determined.

67. *Method second, for the adjustment.* Find two distant objects nearly on opposite points of the compass, as before directed in par. 61. Fix the index at 0, then direct the sight to one of those objects, and cause the reflected image of the other object to coincide with that seen directly, by changing your place as was directed in par. 61. The objects being thus made to coincide, keep the face of the octant upwards, but turn yourself half round, so as to see that object directly which was before seen by reflection, and the other object by reflection which was before seen directly. If they now coincide again, the horizon glass is truly adjusted; if they do not coincide, turn the adjusting lever to bring the object seen by reflection half way towards that seen directly. Proceed as in par. 61, only keep the octant constantly with its face upwards,
and,

and, instead of changing the position of the octant as there directed, change your own position by turning half round. In this method it is best to observe constantly with the left eye. The nearness of the objects is of less consequence in this than in the last method; it will be of none, if the directions for making this adjustment within doors be followed abroad. See par.

72.

68. *Method second, for finding the index error.* Having found two distant objects * nearly on opposite points of the compass, direct the sight to one of them, and turn the index till the other which is behind you coincides with that which is seen directly, and mark the degrees and minutes shown on the limb. Keep the face of the octant still upwards, but turn yourself half round as before mentioned, par. 67. Direct the sight to the other object now before you, and turn the index till the two objects again coincide, and again mark the degrees and minutes shown on the limb. Half the difference of these two arches is the error of the adjustment, and is to be added or sub-

* Here the objects cannot be too far distant.

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tracted according to the rules laid down in the latter end of par. 65.

69. This may be proved by the method in par. 67, just as the first method of finding the index error was proved by the first method of adjusting the horizon glass in par. 66.

70. In the first method the same object is constantly seen directly; and its opposite by reflection; which is a convenience when one object is bright and the other obscure. In the second method you avoid the trouble of holding the instrument with its face downward, and observing with it in an awkward position:

71. You may rectify the horizon glass (or may find the index error) either by changing the position of the octant as in the first method, or by changing your own position as in the second method; but not if you change both the position of the octant and also your own. For whatever objects coincide before *both* these changes are made, the same will coincide after, and that whatever be the position of the horizon glass or index glass.

72. *Method third, for the adjustment; to be used within doors.* Upon an horizontal stand
fix

fix two conical points, whose distance is exactly that between the centers of the index glass and horizon glass; or, which is the same, the distance between the centers of the screws that fasten the index glass and the horizon glass to the plane of the octant. The heads of these screws appear on the back side of the octant; in the center of these you may countersink or turn conical holes to receive the points aforesaid. Provide two small objects that can be easily seen, and that are well defined. A line and plummet hung against two opposite windows by day, or two candles by night, may do. Set the stand nearly in the line joining these two objects. Set o on the Vernier to o on the limb of the octant; fix the index, and then place the octant upon the two points on the stand, making them enter the conical holes in the heads of the screws aforesaid. Turn the stand about in its own place, till you see one of the objects by direct vision through the unsilvered slit in the horizon glass; then look in the silvered part for the reflected image of the other object now behind you. If the direct and reflected objects do not

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coincide, make them do so, either by shifting the place of one of the objects, or by shifting the stand itself across the line joining the objects. When the direct and reflected images are thus made to coincide, then turn the octant half round, placing the screw of the index glass upon the point which before supported the horizon glass; and contrariwise. See if the objects now coincide; if they do, the horizon glass is truly adjusted; if they do not, by means of the adjusting lever bring the object seen by reflection half way towards that seen directly. Then move either one of the objects, or the stand itself, so as to make the two objects coincide perfectly. Turn the octant again half round, placing it on the points as at first. See if the objects, viewed as before, again coincide; if not, repeat this operation till they do, and the horizon glass will be perfectly adjusted.

73. In making this adjustment care must be taken, *first*, that the objects lie in the plane of the octant, and, if lines with a plummet are used, the plane of the octant should be placed nearly level, otherwise the lines, when seen by the octant, cannot be

made to coincide perfectly, but will always seem to cross each other. But *secondly*, when the objects are within a few feet of the octant, particular care must be taken that the reflections be always made from the middle of the glasses, especially the reflection from the index glass. To be sure of this, cut a card to fit the inside of the frame that holds the index glass so as to bed flat against it. Across the middle of this card draw a black line, so that, when the card is applied close to the index glass, this black line may be perpendicular to the plane of the octant. This line, when the card is so applied, may be seen by reflection from the horizon glass. It will be easy, by twisting the stand about and shifting the place of the eye, to make this black line and the object seen directly to coincide, and that in the very middle of the horizon glass. Take away the card, and then, if the image of the other object be made to coincide with that seen directly, it must be by reflection from the middle of each glass: otherwise a thread may be stretched across the middle of the index glass, perpendicular to the plane of the octant, and an-

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other thread in like manner across the middle of the horizon glass. By twisting the stand round, and placing the eye properly, both objects may be made to coincide with each other, and with the threads in the middle of each glass; and then it will be of no consequence how near the objects are to the octant. In this method it is best to observe with the left eye.

74. If in the second method by distant objects, par. 67, the octant was to be placed on a stand in the manner here described, that adjustment would be more perfect, and the best of all the methods here laid down.

75. *Method third, for finding the index error, to be used within doors.* Proceed just as before in par. 72, only instead of turning the adjusting lever to make the object seen by reflection move half way towards that seen directly, turn the index itself. When the two objects are at last made to coincide in both positions of the octant, the index will then show on the limb the index error, which is to be added if this error be shown on the quadrantal arch, and subtracted if it be shown on the arch of excess. See the latter end of par. 65.

76. *Me-*

76. *Method fourth, for the adjustment.* Look out for two objects that subtend an angle of 90 degrees, or near it; adjust the horizon glass for the fore observation by par. 52, and, looking through the fore-sight vane at one of those objects directly, turn the index till the reflected image of the other object coincides with it, and mark the degrees and minutes shown on the limb. Set the index to show that number of degrees and minutes by the back observation: that is, set the index so as to show, by the figures on the limb, the supplement of the number of degrees and minutes before shown in the fore observation, (see par. 24). Then, looking through the back-sight vane, see if the same two objects coincide in the back horizon glass; if not, turn the adjusting lever of this glass till they do coincide, and then this glass also will be adjusted.

77. *Method fourth, for finding the index error.* Having found two objects that subtend an angle of 90 degrees, or a few degrees over or under as before, take the exact angle between these objects by the fore observation, making due allowance for the

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index error according to par. 54 or 56. Again, take the angle between the same objects by the back observation, and the difference of these angles so taken is the index error in all back observations, and is to be added or subtracted as the angle taken by the fore observation (when properly corrected) is greater or less than the angle between the same objects taken by the back observation.

78. The last method supposes the whole octant (or the extent of 90 degrees on the limb) to be an exact eighth part of a circle. The former methods of adjusting the horizon glass depend not at all on the divisions on the limb, and the methods of finding the index error only suppose the divisions to be true for a very small extent on each side of the point from which they begin. In the former methods the observations are made when the index is near 0, where the field of the instrument is the greatest, and the objects easiest found; in this last method the observations are made when the index is near 90 degrees, where the reflections are very oblique, and the field small. It may be remarked too, that the object, which is
seen

seen directly in the fore observation, will be seen by reflection in the back observation. This is an inconvenience, if both objects are not equally bright. By holding the octant with its face downwards in one of these observations, each object will be seen by the same means in both; but it is troublesome to hold the octant in that position.

The method of adjusting the *back horizon glass*, and of finding the error of that adjustment at *SEA*.

79. *To adjust this glass.* Set the index so as to show, on the arch of excess, double the angle of the depression or dip of the sea horizon. * Then holding the octant in a vertical plane, turn the adjusting lever till the edge of the sea, seen in the silvered part of the glass, coincides with the edge of the sea seen through the transparent slit of the horizon glass. Fix the adjusting lever in that place, and the octant is adjusted.

80. In this position of the octant, the object seen by reflection, which is behind

* See a table of the depression or dip of the horizon of the sea, among the tables belonging to the *Nautical Ephemeris*, published by order of the Commissioners of Longitude.

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the observer, will appear to him to be before and inverted. Thus in the present instance the water will appear above and the sky below. If the two edges of the sea (that seen directly and that seen by reflection) cannot be made to touch all along, but cross each other, it is a sign that the octant is not held in a vertical plane. Its position must be altered, till the two edges of the sea so seen are parallel, and then the adjusting lever will bring them to touch all along.

81. If the index glass and horizon glass are too near each other, the head of the observer will be in the way. The best construction of this octant will not allow the observer to have his hat on, or any thing high upon his head.

82. *To find the index error.* Hold the octant in a vertical plane, the index glass being uppermost as usual. Turn the index till the two edges of the sea (one seen directly and the other by reflection) meet each other and touch all along; then mark the degrees and minutes shown by the index on the limb. Invert now the octant, so that the arch may be uppermost, but the
index

index glass and center of the instrument lowest; holding the arch with your left hand, and supporting the lower part of the octant with your right, yet so as not to touch or shade any of the glasses with your hand. Let the face of the octant be turned towards you, but keep the plane of the octant vertical. Incline now your head sidewise till you bring your right eye down to the sight vane, and, looking through it, move the index by the left hand, till the two edges of the sea (that seen directly and that by reflection) agree as before, and again mark the degrees and minutes shown on the limb; half the difference of these two arches is the index error. * The index error so found is to be added or subtracted according to the directions in par. 65.

83. That rule given in par. 65 holds good in taking distances between the heavenly bodies, or in taking angles between land objects; but in one sort of observation, namely, *taking the sun's altitude by the back observation*, the index error, found as before, follows a contrary rule. It is to be sub-

* One fourth of the sum of these two arches is the dip of the sea horizon.

tracted

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tracted from the sun's altitude, when the portion of the quadrantal arch exceeds the portion of the arch of excess: if contrariwise, this index error is to be added. The reason of this is because the figures engraved on the limb immediately show the sun's altitude in the back observation, just as they do in the fore observation. See par. 25.

84. It may be of some use to suspend the octant by a line tied round the arch. This line bearing the weight of the octant, it may be more easily guided into the proper position by the right hand, while the index is set by the left. If the observer chuses to make use of his left eye rather than the right, then the back of the octant must be held towards him, and whatever was before directed to be done with the right hand must now be done with the left, and contrariwise.

It must be confessed that this method of finding the index error is rather troublesome. Possibly that before described, par. 75, may be put in practice on shipboard. If no other way can be practised, the mariner may take the sun's meridian altitude by the back observation immediately after he has
com-

completed his fore observation for the latitude. When all the necessary corrections are made, except that for the index error sought, then compare the sun's true altitude deduced from the fore observation, with the like result from the back observation: Their difference may be considered as arising wholly from the wrong position of the back horizon glass, and may be applied accordingly to correct this latter observation of the sun's altitude. This correction must be applied contrariwise in taking distances between the heavenly bodies, as was before remarked, par. 83.

To adjust the place of the sight vanes, and other parts of the octant.

85. We seldom find the sight vanes set exactly in their true place. Their true place is that which gives the largest field. This depends on the placing of the index glass upon the index, and the point of the limb from whence the divisions take their beginning. We shall show *first* the method of finding the true place of each sight vane, supposing the index glass to be already fixed on the index, and the point from whence the
the

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the divisions are to commence to be already marked upon the limb. *Secondly*, To find the point of the limb from whence the divisions ought to take their beginning, supposing the place of the vane determined, and the index glass fixed on the index. *Thirdly*, How to fix the index glass properly upon the index, supposing the sight vane already placed, and the point determined from whence the divisions are to begin.

86. For the first, adjust the index glass by par. 44. Set the index to the beginning of the divisions. Find the place of the sight vane nearly, which you will easily do by moving the eye about till you can see some object by the double reflection. Adjust now the horizon glass exactly by par. 49, 52; then shift the sight vane till the center of the index glass is seen by reflection from the middle of the horizon glass, and that will be the true place of the sight vane.

87. You may use the adjusting tool, fig. 10, see par. 44, by way of a moveable sight vane, and you may stretch threads across the index and horizon glasses to distinguish their middle, as was proposed par. 73. But the best way is as follows,

First

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First turn the index to 90 degrees, so that the whole frame of the index glass may be seen as an object by reflection from the horizon glass: then turn the index gradually back again, and at the same time shift the place of the sight vane till the reflected image of the frame of the index glass just fills up the whole area of the horizon glass; the two ends of the frame of each glass apparently coinciding; and the sight vane will then stand in its true place.

The place of the sight vane for the back observation ought to be adjusted very exactly, because the field is generally very small.

88. For the second point proposed, Turn the fore horizon glass by the adjusting lever till the center of the index glass is seen through the fore-sight vane, by reflection from the middle of the horizon glass, or till the reflected image of the frame of the index glass can be made just to fill up the area of the horizon glass as before. Turn now the index till the reflected image of some distant object coincides with that object itself as described in par. 52, and that will be the place of the index at the beginning of the divisions. This found, the point
from

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from whence the divisions ought to commence may be easily marked on the limb. This point might have been determined from the assumed place of the back-sight vane, but it is best to determine it from the place of the fore-sight vane, and then to find the place of the back-sight vane from this point so determined.

89. The common octants are, I believe, made by one workman, and then sent to another to be graduated; perhaps without marking out where the divisions should begin. If he who graduates the arch should happen to begin at some distance from the proper point, (which depends upon what the former workman has done) half the field may be lost when the index stands at 90 degrees.

90. For the last point proposed, set a pin upright in the center of the index. Turn the fore horizon glass by the adjusting lever till this pin is seen by reflection from the middle of the horizon glass. Set the index to the beginning of the divisions; then, removing the pin, place the index glass upon the center of the index. Turn the index glass round that center upon the index till the

OF CONSTRUCTING AN OCTANT. 63

the reflected image of some distant object coincides with that object itself seen directly as before, and that will be the true place of the index glass upon the index.

Rules for constructing an Octant.

91. The foregoing rules suppose some circumstances in the octant already determined on which those rules are founded. No very precise rules can be given for setting out an octant intirely from the beginning, but the general principles upon which it ought to be constructed may be laid down. Now one principle is, that very oblique reflections from the mirrors ought to be avoided. For this reason the center of the horizon glass should be placed, as near as possible, in a line drawn through the center of the index glass, and perpendicular to its surface when the index stands at the beginning of the divisions. I say *as near as possible*; for the horizon glass cannot be placed in this very line. In such a case it would stand between the index glass and the object which should be seen by reflection, and so screen this object from the sight. The horizon glass itself ought not to be seen by
the

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the double reflection, and yet should be but just out of sight. Accordingly, when the index is turned a little way off the limb, the fore horizon glass should be seen by its own reflection, but not while the index is on the limb.

92. The place of the fore horizon glass being once determined, its position, and the position of its axis of vision, follows of course. The former must be parallel to the index glass when the index stands at the beginning of the divisions: the latter, namely the axis of vision, will be found by supposing the line, joining the centers of the two glasses aforesaid, to be a ray incident on the horizon glass; for the reflected ray is the axis of vision, agreeably to par. 18 and 86. *

93. To place the back horizon glass properly is a critical business. Care must be taken that the fore horizon glass does not intercept the rays passing between the index glass and back horizon glass; for which rea-

* It may be convenient so to place the index glass upon the index, that its plane continued, may cut the index down the middle. The weight of the index, when turned off the limb to adjust the index glass, (par. 44.) will not then alter the inclination of that glass to the plane of the octant.

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son this glass cannot stand in the line drawn through the centers of the other two glasses, but on one side that line. And it must stand the same way from that line, that \odot on the limb stands from 90; otherwise this (back) horizon glass would intercept the view of some of the objects which should be seen by reflection in the fore horizon glass. The nearer the back horizon glass stands to that line, the more oblique will be the reflections at this glass, and the less oblique the reflections at the index glass. The latter is of most consequence, because of the great length of the index glass, necessary to take in a moderate field, when these reflections are very oblique: therefore though the back horizon glass should stand on one side the line before mentioned, to clear the fore horizon glass, yet it should be but just clear of it. The back horizon glass must be so far distant from the index glass that the head of the observer may not be in his own way; and no more: for the greater this distance the less will be the field. The back horizon glass always stands at right angles to the fore horizon glass, whence the position of its axis of vision may be

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deter^a

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determined as before, par. 92, at the end. *

94. The foregoing rules being considered, the plan of a good octant will be found to lie

* It may not be amiss to set down the following mathematical rules, omitting their demonstration.

1. In the fore observation, the angle between the ray incident on the index glass and that ray after two reflections, is double the inclination of the faces of the speculums: and this is the angle shown on the limb, when the glasses are perfectly adjusted. This is likewise the angle which the (doubly) reflected image of any object makes with that object itself: and if that reflected image coincides with some other object seen directly through the transparent part of the horizon glass; then this is the angle which the object seen by reflection and the object seen through the glass would subtend to the naked eye, placed at the intersection of the incident ray with that ray after two reflections.—These different expressions of the same quantity are here noted once for all.

2. In the back observation, the angle between the ray incident on the index glass and that ray after two reflections is double the supplement (to 180 degrees) of the angle which the faces of the speculums make with each other. And this is the angle shown on the limb if it be read off by the rule in par. 24.

3. The angle which the line, joining the centers of the index glass and fore horizon glass, makes with a perpendicular to the index glass, when the index stands at the beginning of the divisions, is equal to the angle of incidence on the index glass in that position; and this is the constant angle of incidence on the fore horizon glass.

4. The angle of incidence on the index glass, when the index stands at 90 degrees, exceeds the former angle by 45 degrees.

5. In the back observation, the angle of incidence on the index glass is greater than the angle of incidence in the fore observation (the index standing at the same place) by the angle which the centers of the two lesser mirrors subtend at the center of the great one.

6. The angle of incidence on the back horizon glass is constantly the complement (to 90 degrees) of the sum of the angle of incidence on the fore horizon glass, and the angle
which

lie within narrow limits. The design first published in the Transactions, Nr. 420, and generally followed ever since, will scarce admit of any alterations. I have seen indeed, in the hands of an optician, an octant, which, besides the great moveable mirror, had two small mirrors fixed in the middle of its plane. This was represented as a *new invention*, but it seemed not to differ essentially from the old one of Hadley's. The number of glasses in both instruments is the same. The rays proceed in the very same manner *relatively to the glasses*, and the glasses stand in the very same position *relatively to each other*, though not to the parts of which the instrument is composed. This octant was much celebrated because it would take an angle of 180 degrees; Hadley's does the same. Indeed Hadley seems to have understood the theory so perfectly, and to have consulted practical convenience so fully, that further improvements in the construction of this instrument can hardly be expected.

which the centers of the two lesser mirrors subtend at the center of the great one.

7. The angle which the two axes of vision (for the fore and back observation) make with each other, is equal to that which the two lesser mirrors subtend at the center of the great one.

95. It may be of some use to give here the proper distances between the several glasses, for any octant large enough to admit of the back observation, and likewise the several angles of incidence at each mirror.*

	Inches
Distance of the center of the index glass from the center of the fore horizon glass	4 $\frac{1}{4}$
Center of the index glass from the center of the back horizon glass	6
The two horizon glasses from each other	2
Length of the index glass	2 $\frac{1}{4}$
of the fore horizon glass	1
of the back horizon glass	1 $\frac{1}{4}$
	D M
The angle which the two horizon glasses subtend at the index glass	12 30
Angle of incidence on the index glass in the fore observation, the index standing at the beginning of the divisions. Also the constant angle of incidence on the fore horizon glass	15 30
Angle of incidence on the index glass in the fore observation, the index standing at 90 degrees -	60 30
The former of these angles in the back observation	28 00
The latter of these angles in the back observation	73 00
The constant angle of incidence on the back horizon glass	62 00

Of the Motions of the Reflected Image.

96. We shall here describe the motions of the reflected image, while the octant is turned round some of the principal lines both perpendicular and parallel to its plane.

* The angles of incidence are reckoned from the perpendicular.

97. And first suppose the octant to be turned round the common intersection of the two reflecting planes, which will be a line perpendicular to the plane of the instrument, and the instrument itself will then move in its own plane round that axis. In this case the reflected image of any object will be perfectly at rest. Let this image coincide with any other object seen directly, and they will constantly stick together and be united notwithstanding this motion of the octant; and that whether the object be near or distant, and whatever be the inclination of the reflecting planes to each other. This is the grand principle demonstrated by *Hadley*, *Phil. Trans.* Nr. 420. Almost all other writers have only transcribed his paper word for word. *

98. Se-

* *Smith's Optics.* *Observaciones Astronomicas &c.* por *D. Jorge Juan y D. Antonio de Ulloa.*

If two looking-glasses be joined together so as to form an acute angle, and an object be placed between them, a spectator will see a number of images of that object in each glass, all placed in the circumference of a circle whose center is in the common section of the two glasses, and whose radius is the distance of the object from that section. These images are formed by an odd and an even number of reflections alternately. Thus let the glass into which you look be called *A*, and the other *B*; then the first image is formed by the reflection of *A* only. The next image is formed by two reflections, first from *B* and then from *A*. The third image is formed by three reflections; first from *A*, then from *B*,

98. Secondly. Suppose the octant turned round some other axis parallel to the former and therefore perpendicular to the plane of the instrument. In this case distant objects, seen by reflection, will have no motion at all; the reflected image of a near object will have a small motion parallel to the plane of the octant, but will keep its position parallel to itself.

99. Let now the axis of motion be parallel to the plane of the octant, and first suppose the axis of vision to be the axis of motion. Then in the fore horizon glass if the reflected image of any object, whether

and then from *A* again, and so on. If now the two glasses, keeping their inclination, be turned round their common section as an axis; these images will alternately move and be at rest. All the images formed by an odd number of reflections will move round their common center with double the angular motion of the glasses, whilst the images formed by an even number of reflections will remain perfectly fixed.

This experiment is not only entertaining, but suggests an easy proof of the property of the octant here mentioned. For when the number of reflections is even, the distance of any of those images from the original object, measured along the circumference of the circle in which they all stand, is equal to the arch (of that circle) intercepted by the reflecting planes, multiplied by the number of reflections. Therefore as the quantity of the intercepted arch is not changed by the common motion of the planes round their angular meeting, the distance of none of these images from the fixed object will be altered, and consequently they will all remain at rest; and among them, that formed by two reflections only.

near

near or distant, be made to coincide with that object seen directly, it will ever remain at rest; the direct and reflected images of that object being perfectly united in every position of the octant. If the reflected image be separated from the object by moving the index, the image will then describe a circle whose center is that object itself, and it will move the same way and with the same angular velocity that the octant does; but the reflected image will keep its position parallel to itself and to the original object.

100. Hence, in the vibratory motion of the octant round the axis of vision, the reflected image will describe an arch of a circle (to and fro) convex towards those objects that lie at a greater angular distance from the original object than its reflected image does, that is such as are further distant than the angle shown on the octant.* In this vibratory motion, the image keeps its position parallel to itself, and it crosses the plane, passing through the axis of motion and the object, at right angles,

* See the note on par. 93. Nr 1.

101. In the back horizon glass, when the index shows 180 degrees and the glasses make a right angle, if the octant be turned round the axis of vision, the reflected images of the several objects behind you will describe concentric circles round the axis of vision, and the common center of these circles will be that object before you, to which the axis of vision is directed, or that point of the reflected image which coincides with this object. The whole reflected image therefore changes its position just as the sails of a windmill do. The motion of this image is the same way with that of the octant, but has double its angular velocity. When the plane of the octant is horizontal, this reflected image is upright; when the octant has been turned round 90 degrees, so as to come into a vertical plane, the reflected image will then be turned round 180 degrees, that is it will be inverted.

102. If the index be removed from 180 degrees (so as to increase the angle between the speculums) and the octant be made to vibrate about the axis of vision, the reflected image will describe the arch of a circle convex towards such objects as are further distant

distant from the original object than the angle shown on the octant, reckoning that distance the same way that the image is removed from the object. The image in this case will continually change its position, and it will cross at right angles a plane passing through the axis of motion and the original object.

103. Let the octant be made to vibrate about an axis a little inclined to this axis of vision, but in the same plane as before; for instance, let it be made to vibrate about the axis of vision belonging to the fore horizon glass. Let the index stand at 180 degrees as before, the reflected image, seen in the back horizon glass, will describe an arch of a circle whose center will be that object to which the axis of motion is directed. If the index stands at any other place, the reflected image will describe an arch of a circle convex the same way as described before, par. 102. In both these cases the image changes its position as well as its place. In these vibratory motions the angular velocity of the image depends on the place of the index. At 90 degrees it is nearly equal to that of the octant, at 180 degrees nearly double.

104. Let now the axis of motion be perpendicular to the axis of vision, but parallel to the plane of the octant, and let the index stand at 0 in the fore observation, that is at 180 in the back observation. Then in the fore horizon glass the reflected image will have no motion. In the back horizon glass the reflected image will turn round the axis of motion with double the angular velocity of the octant. In this last case the image will keep its position parallel to itself. Thus if the octant be held in a vertical plane, and the image of a steeple behind you be seen in the back horizon glass; then, if the octant be turned about the forementioned axis, that image will be carried round the horizon. The steeple will indeed appear inverted, according to par. 101, but will keep that position, and move parallel to itself round the horizon.

105. If the index stand at 90 degrees, and the octant be turned about the forementioned axis, the reflected image will be carried round that axis with the angular velocity of the octant, and the position of the image will likewise be changed with the same angular velocity. Thus let the plane of the
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the octant be held horizontal, and the index set at 90 degrees; let the octant be turned round an axis parallel to the horizon, but perpendicular to the axis of vision. Then the reflected image of a steeple, whether seen in the fore or back horizon glass, will be carried round the axis of motion, so that when the octant has described 90 degrees, and is come into a vertical plane, the steeple will appear either in the Zenith or Nadir (according to the direction in which the octant is turned) but instead of being erect the steeple will appear parallel to the horizon. In intermediate places of the index the image will partake more or less of those motions, which it has when the index is at either extremity of the arch.

106. The several axes round which the octant is supposed to vibrate in the last seven paragraphs are all supposed to be parallel to the plane of the octant, but to pass through the axis of vision, intersecting it at different angles. If, instead of this, we suppose the several axes of motion to lie in the plane of the octant itself, and to be respectively parallel to the

several

Several axes of motion in the former case, then the apparent motion of the reflected image in each case will be the same as before, provided the original object be not too near the octant.

THE APPLICATION OF THE OCTANT TO SEVERAL PURPOSES.

General rules in making Observations.

Directions how to hold the instrument.

107. The best way of holding the octant is to support its weight chiefly by the right hand, and to reserve the left to govern the index. Place the thumb of the right hand against the edge of the octant, under the swelling part on which the fore-sight vane stands, and extend your fingers across the back of the octant so as to lay hold on the opposite edge, placing the fore finger above and the other fingers below the swelling part on which the fore horizon glass stands. Thus you may support the instrument conveniently by the right hand only, especially when in a vertical position. By resting the thumb of the left hand against the side, or the fingers against the middle bar, you may draw the index either to or from you gradually; but if the index is
fur-

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furnished with an adjusting screw you will be able to set it more exactly; still more so if the octant be supported by a stand. In taking the distance of the moon from the sun or stars to find the longitude at sea, the weight of the instrument is frequently supported by a staff, the bottom of which rests on a belt, or in the waistcoat pocket of the observer. At the top of this staff is a ball and socket, upon which the instrument is fixed, and by means whereof it can be readily put into the proper plane for observing.

The foregoing directions relate to the fore observations. You must hold the instrument in like manner for the back observation; only here you should support the instrument by the left hand, and govern the index by the right.

Of the proper plane in which all observations are to be made.

108. In all observations care must be taken to hold the octant in the proper plane, namely the plane which passes through the eye of the observer and the two objects; that is, in astronomical observations, in the plane

of a great circle passing through the two objects. It is not easy to know in all cases when the octant is exactly in that plane. One necessary caution is to make the union of the two objects (or, as we may more properly call it, the union of direct and reflected images) in the proper part of the horizon glass, and then, if a tube of considerable length be used to direct the sight, the position of the octant cannot be very faulty. Now the union of the direct and reflected images in the horizon glass, ought to be made in a line parallel to the plane of the octant, but passing through that point of the glass to which the axis of the tube is directed. If a telescope be used, this union should be made in a line drawn cross the middle of the field parallel to the plane of the octant. And a telescope is of use not only to distinguish the contact or union of the images more perfectly, but likewise to find by the place of this union in the field, whether the octant be held in the proper plane.

109. Such a stand as was mentioned in par. 31. is likewise of service for this purpose. Let the plane of the octant be set
per-

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perpendicular to the axis QQ Fig. 4, and be fixed by the screws TT . Let the whole be turned round the axis QQ ; if you then see both objects successively through the tube or telescope by direct vision, the octant must of course be in the proper plane.

110. Making the octant vibrate round the axis of vision (or any other axis parallel to it) affords a way of knowing when the octant is held properly, as will be explained hereafter.

111. Another method peculiar to the back observation, but very useful in taking horizontal angles at land, is to observe whether the part of the horizon seen by reflection coincides with, or is parallel to that part of the horizon which is seen directly. If the two horizons cross each other, the plane of the octant is not level. A similar circumstance, and equally useful at sea, will be taken notice of in its proper place.

112. In order to judge of the importance of holding the octant in the proper plane, take off the sight-vane, place the eye either above or below its proper height (to which the sight-vane guides it) so as to look at
the

the middle of the horizon glass in a direction oblique to the plane of the octant. By inclining the octant, and moving the index, you may cause the images to be united in the middle of the horizon glass, notwithstanding you look obliquely. Fix the index and note the angle. Put on the sight-vane, and, holding the octant in the proper plane, you will find the images will not now be united. Bring now the images together, and again note the angle, and you will find that the angle in the former case will exceed the angle in the latter by many minutes, in some cases by a whole degree. The same may be done with the sight-vane in its place, by looking obliquely upwards so as to get the union of the images to the top of the horizon glass; provided one of the objects be bright enough to be seen by reflection from the transparent part of the horizon glass.

Of the vibratory motion to be given to the octant at the time of observation.

113. When the images are brought nearly into contact, it is useful to swing the octant to and fro round the axis of vision,

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or

or some line parallel to it. If you hold the octant as before directed, par. 107, you may easily turn it about between the fingers and thumb of the right hand. By this means the reflected image will be made to vibrate backwards and forwards across the plane of the instrument. It is often easier to distinguish whether the images coincide in passing one another, than by bringing them together while the octant is held steady. Thus in taking the distance of two stars, this motion of the reflected star will make on the eye the impression of a line of bright fire, which impression will remain some time. It is easy to judge whether this line passes through or at a distance from the other star seen directly.

114. In taking distances of the sun or moon, from the sea horizon, from the stars, or from each other, this vibratory motion of the reflected image of the sun or moon serves to distinguish that part of their limb which is the nearest or remotest from the other object: it is always the distance of one of these two points which is to be measured. This will be further explained when we treat of these observations in particular.

115. Lastly, this vibratory motion is of some use to find whether the octant be held in the proper plane. The traces of the arch, described by the reflected image in its vibratory motion, will remain in the mind even when the octant is at rest, and this arch ought to be perpendicular to the plane of the octant, when it is in that position in which the images appear united, or pass by each other in their vibratory motion.

116. In strictness the two images cannot appear to coincide or be united perfectly, unless one of the objects be bright enough to be seen by *reflection from* the transparent part of the horizon glass, while the other object is seen *directly through* the very same point of that glass. When this is not the case, all that can be done is to make the two images meet and as it were join at the separation of the silvered and transparent parts of the horizon glass. If the octant be turned about to and fro round the axis of vision, so as to cause a vibratory motion of the reflected image, it will then be easy to judge whether the line, which that image describes, would, if produced, go

through the object seen by direct vision, or pass at a distance.

Astronomical Observations at SEA.

To take the altitude of the sun at sea by the fore observation.

117. The sun is generally bright enough to be seen by reflection from the unfiltered part of the horizon glass; uncover therefore the uppermost hole in the sight-vane, or, if the octant is furnished either with a tube or a telescope, raise it so as to be opposite to the transparent part of the horizon glass; put likewise the red glasses into their proper place. Hold now the octant in a vertical position, and, turning yourself towards the sun, direct the sight to that part of the sea horizon which is under the sun. Then begin to move the index from 0 on the limb till you see the red image of the sun come down towards the horizon. If you meet with any difficulty in finding the (red) reflected image of the sun, turn back the
red

red glass, and the sun's image will then be so bright that you cannot miss it: when found, return the red glass into its proper situation. Having brought the sun's image almost down to the horizon, then with your left hand swing the octant backwards and forwards, turning it round the line passing between the fingers and thumb of the right hand, and the image of the sun will seem to describe the arch of a circle whose convex side is downwards. Move the index till the lower edge of the sun's image just grazes upon the horizon, when it is at the lowest point of the arch apparently described, and then stop the index. Read off the degrees and minutes shown by the index, and call this *The observed altitude*, to which the following corrections must be applied.

118. *First*. The index error——To be added or subtracted by the directions in par. 54.

119. *Secondly*. The dip of the sea horizon——To be found in the tables belonging to the *Nautical Ephemeris*, page 14. * This is to be subtracted.

* Published by order of the Commissioners of Longitude.

120. *Thirdly.* The sun's semidiameter—
—This is to be found in the third page
belonging to every month in the Nautical
Ephemeris, and is to be added.

These three corrections properly made,
you will have *The apparent altitude of the
sun's center.* To this apply two other cor-
rections, viz.

121. *Fourthly.* The refraction—To be
found in the aforesaid tables, page 2, and
is to be subtracted.

122. *Fifthly.* The sun's parallax—To
be found in the additions to the Nautical
Ephemeris for 1772, page 37, and is to be
added.

And thus you will have *The TRUE
ALTITUDE of the sun's center* (as seen
from the center of the earth) from whence
you may calculate either the hour of the
day, or the latitude of the place.

123. When great nicety is not required,
you may suppose the sun's semidiameter
to be always 16 minutes. And the last
correction (viz. for the sun's parallax) is
so small that it may generally be omit-
ted.

To take the altitude of the sun by the back observation.

124. Having removed the red glasses into their proper place, hold the octant in your left hand, and in a vertical position. Direct the sight to the part of the horizon opposite to the sun: then, applying your right hand to the index, move the index gradually from 0, till you bring the red image of the sun *up* to the horizon, so that the *apparent* lower edge may stand upon the horizon. * If you now swing the octant between the fingers and thumb of your left hand, the image of the sun will describe an arch of a circle whose convex side is downwards. Move the index till the *apparent* lower edge of the sun's image just grazes upon the horizon in the lowest point of that arch, and then read off the degrees and minutes as before; see par. 25. Let this be called the *observed altitude*, to which apply the following corrections.

125. *First.* The index error——To be added or subtracted according to the directions in par. 83.

* The apparent lower edge is not the real lower edge. See par. 80 and 101.

126. *Secondly.* The dip of the horizon—
—To be found as before, par. 119, but
must now be added.

127. *Thirdly.* The sun's semidiameter—
—To be found as before, par. 120, but
must now be subtracted.

These three corrections give the *apparent altitude* of the sun's center, to which apply the other two corrections as before, par. 121 and 122, and you will have the *true altitude* of the sun's center.

128. Instead of making the octant vibrate between the finger and thumb of the left hand, you may use this motion. Keep the octant steady in your hands, but turn yourself round upon your heel; or rather, keeping your body upright, twist yourself from the right to the left, and back again, so as to turn the octant round a vertical axis. Observe whether the image of the sun continually touches the horizon during this motion. If it does, then the plane of the octant is held exactly vertical. If in that motion the sun's image crosses the horizon, the plane of the octant is not vertical, and you must shift its position till you can make the limb of the sun constantly glide along
6 the

the horizon, while you turn yourself round as before.

Either of these motions serve as a proof that the octant is held in the proper position, just at the very time when the index was set. The latter is that generally recommended.

To take the altitude of the moon.

129. The moon's altitude may be taken either by the fore or back observation exactly in the same manner as the sun's altitude, only here you must bring that edge of the moon into contact with the horizon which is round and well defined, whether that be the upper or under edge. The corrections to be applied to the *observed altitude* are as follows:

130. *First.* The index error. To be applied as before directed, par. 54. for the fore observation, and par. 83. for the back observation.

131. *Secondly.* The dip. To be found as before, par. 119, and subtracted in the fore observation, but added in the back observation.

132. *Thirdly.* Semidiameter. To be found
in

in the Nautical Ephemeris for every noon and midnight at Greenwich. If very great accuracy is required, this semidiameter must be corrected for the intermediate time and for the altitude by the rule in page 152 of the tables belonging to the Nautical Ephemeris. The semidiameter, so found, is to be added in the fore observation, and subtracted in the back observation, if the moon's apparent lower limb was observed; that is, if the lower edge of the moon's image, as seen by reflection in the octant (through plain sights) was brought into contact with the horizon; but if the upper edge (as seen by reflection in the octant) was brought into contact with the horizon, then this semidiameter is to be subtracted in the fore observation, and added in the back observation: and thus you will have the *apparent altitude* of the moon's center.

133. *Fourthly*. Refraction. To be found and applied as before, par. 121.

134. *Fifthly*. Parallax. The moon's *horizontal* parallax for every noon and midnight at Greenwich is to be found in the Nautical Ephemeris. When nicety is required, this must be corrected for all intermediate times by the rule in page 152 of the tables

tables belonging to the Ephemeris. The moon's *horizontal* parallax being thus obtained, the moon's parallax *in altitude* may be found therefrom by page 3 of the tables aforesaid, and is to be added. Otherwise in page 52 you will find a table which gives the correction both for refraction and parallax in one. This added to the apparent gives the true altitude.

To take the altitude of a star by the fore observation.

135. Set the index at 0, and, holding the plane of the octant vertical, direct the sight to the star, and at the same time look for the reflected image of the star in the silvered part of the horizon glass. Stir the index a little, which will separate the reflected image from the direct image; the former will be easily distinguished from the latter by its motion when you stir the index. Continue to advance the index, and at the same time follow the reflected image of the star with your eye, directing the sight lower and lower, and changing the position of the octant as the image of the star descends, till you have brought it down to the horizon: the index will then show the observed altitude

altitude of the star. It is necessary that you should always keep sight of the reflected image of the star, that you may not mistake the reflected image of some other star for the image of that whose altitude you would take. But if you know its altitude nearly, you may then set the index at first to that altitude, and, holding the octant vertical, direct the sight to the part of the horizon just under the star, and you will be sure to find its reflected image near the horizon. By moving the index a little you may bring it to touch the horizon as before.

136. The corrections to be applied to the observed altitude of a star are *first* the index error, as in par. 54. *Secondly* the dip, as in par. 119. These two give the apparent altitude. *Thirdly* the refraction, as in par. 121, which gives the true altitude. The fixed stars have neither semidiameter nor parallax.

To take the altitude of a star by the back observation.

137. In this case you must know the altitude nearly, or it will be very difficult to

to find the reflected image of the star. This known, set the index to the supposed altitude; hold the octant vertical; direct the sight to the part of the horizon opposite to the star. When you have found the reflected image, bring it to touch the horizon by moving the index as before.

138. The altitude so observed must be corrected, *first* for the index error by par. 83. *Secondly* for the dip by par. 126. *Thirdly* for refraction by par. 121.

The sea horizon, I believe, can rarely be seen at night, unless perhaps by the twilight or moon-light, so that altitudes of the stars so observed cannot be much relied on.

To take the distance of the moon from the sun, in order to find the Longitude at sea.

139. In the Nautical Ephemeris you have the central distances of the sun and moon for every three hours of time at Greenwich on such days as this method is practicable. From these distances in the Ephemeris you are to compute roughly the distance between their nearest limbs at the time of observation as directed in page 165 and 166 of the Ephemeris. If that distance be under 90 degrees,

degrees, you must use the fore observation; if above 90 degrees, the back observation.

140. The moon must be viewed directly through the unfiltered part of the horizon glass, but the sun by reflection, and, if it be bright, by reflection from the unfiltered part of the glass. Hence if the sun be to the left hand of the moon, the octant must be held with its face downward in the fore observation, but with its face upwards in the back observation. If the sun be to the right hand of the moon, the octant must be held with its face upwards in the fore observation, but downwards in the back observation. * One or both of the red glasses must be let down, as the sun is more or less bright.

141. Set the index to the distance between the nearest limbs of the sun and moon computed roughly as before, and, placing the face of the octant by the foregoing rules, direct the sight to the moon. Then put the plane of the octant into *the plane of the*

* When an octant is furnished with a ball and socket, to be fixed either against the face or the back of the octant, it may then be held conveniently either way. When that is not the case, it may be worth while to have a *left-banded* octant made; that is, one in which all the parts are reversed as to right and left. Such an one must be held with the face upwards, when a common one is to be held with the face downwards.

*great circle passing through the sun and moon, according to par. 108: that is, put it into such a position, that if you look along the surface, or rather edgewise against the octant, it may seem to make a line passing through the sun and moon. The knack of doing this is to be got by practice only. Swing now the octant between the finger and thumb, and the red image of the sun will pass by the moon to and fro, and so near that you cannot miss seeing it. Their nearest edges must then be brought into exact contact. And here observe, *first*, that the images of the sun and moon must touch only at their external edges; the body of the sun must not (in this vibratory motion) pass over, or ever be upon the body of the moon. *Secondly*, the edge of the sun must touch the enlightened edge or round part of the moon, not the part from the sun which is either hollow or gibbous, and always ill defined. * These two cautions observed, move the index till (by giving the octant a vibratory motion*

* From new moon to the full, the enlightened edge of the moon is turned towards the West. From the full moon to the new, the enlightened edge is turned towards the East. This is the case when the moon is seen directly; but the contrary if it be viewed through a telescope that inverts the objects.

as before) the edge of the sun just brushes the edge of the moon as they pass by each other. The index then shows on the limb the *observed distance* of the nearest edges of the sun and moon, to which apply the following corrections.

142. *First.* The index error, as in par. 54 and 65.

143. *Secondly.* Semidiameter. The semidiameter of the sun for every sixth day, and of the moon for every noon and midnight, at Greenwich, are to be found in the Nautical Ephemeris; from whence their semidiameters at the time of observation are to be computed by the rule in page 152 of the tables belonging to that Ephemeris. The sum of these two semidiameters is to be added.

These two corrections, properly applied, give the *apparent distance* of the centers of the sun and moon.

144. *Thirdly.* This apparent distance must be corrected for refraction and parallax, and then you will have the REDUCED DISTANCE, or TRUE DISTANCE, of the centers of the sun and moon, as seen from the center of the earth. To make this last correction, two sets of tables, with directions how

how to use them, are given among the tables belonging to the Nautical Ephemeris. The former set of tables (by *Mr. Lyons*) gives the correction for refraction and parallax separately. The latter (by *Mr. Dunthorne*) gives both corrections in one. A set of rules for calculating this correction is given (by *Mr. Witchell*) in the Nautical Ephemeris for 1772. In this Ephemeris are likewise some corrections of the former tables, and additions to the directions before given, which ought not to be neglected. Lastly a set of tables for finding this correction by inspection only, is now in the press at Cambridge under the direction of one of the professors of astronomy, and may be expected shortly by the public.

145. From the reduced distance and time of observation, the LONGITUDE of the place may be computed by the rules and tables belonging to the Nautical Ephemeris.

To take the distance of the moon from such stars as are selected in the Nautical Ephemeris, for finding the Longitude at sea.

146. The central distances of these stars (like those of the sun) are given in the

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Nautical Ephemeris for every three hours of time at Greenwich, from whence their distance from the enlightened edge of the moon may be roughly computed as before. None of these distances exceed 90 degrees, and therefore the fore observation only is to be used.

147. The star must be viewed directly through the transparent part of the horizon glass, but the moon by reflection: therefore when the star is to the left hand of the moon, the octant must be held with its face upwards; when the star is to the right hand of the moon, the octant must be held with its face downwards. The fainter of the red glasses must be let down, which so far takes off the glare of the moon, that the star may be seen actually *upon* her reflected image.

148. Set the index to the distance of the star from the enlightened edge of the moon computed roughly as before, and, placing the face of the octant by the foregoing rules, direct the sight to the star. Then put the octant into the proper plane, so that, if seen edgewise, it may seem to make a line passing through the moon and star. Swing
now

now the octant between the finger and thumb of the right hand, and the red image of the moon will pass by the star to and fro, and so near that you will see it; and this is a proof that the sight is directed to the right star. The enlightened edge of the moon) (whether East or West) must then be brought into contact with the star by moving the index. * You will know this by the vibratory motion of the moon's image before mentioned. For the star should just graze the edge, without entering at all within the body of the moon. The index will then show the *observed distance* of the star from the enlightened edge of the moon, to which apply the following corrections.

149. *First*. Index error, as in par. 54.

150. *Secondly*. Semidiameter of the moon. To be found in the Nautical Ephemeris for every noon and midnight at Greenwich, and from thence to be computed for the time of observation by the rule in page 152 of the tables belonging to that Ephemeris. After the new and before the full moon, this semidiameter is to be subtracted if the star be East, and added if the star be West of the

* See the note on par. 141.

moon. After the full and before the new moon this semidiameter is to be added, if the star be East, and subtracted if the star be West of the moon.

These two corrections give the *apparent distance* of the moon's center from the star.

151. *Thirdly*. This apparent distance must be corrected for refraction and parallax, in the same manner as the apparent distance of the sun was corrected par. 144. This will give the *reduced* or *true distance* of the center of the moon from the star, as seen from the center of the earth.

From the reduced distance, and time of observation, the *Longitude* may be computed by the rules and tables belonging to the Nautical Ephemeris.

152. It will save some trouble, and may serve the purpose of finding the reflected image of the sun or moon in the horizon glass, if you only set the index to the central distance as set down (in the Ephemeris) for the nearest three hours, without correcting it to the intermediate time by a rough computation, as before directed.

153. If you have any doubt whether the sight be directed to the proper star, set the

index to the supposed distance as before, and then find the reflected image of the moon in the horizon glass in the following manner. Having turned back the red glass, hold the octant in any plane passing through the moon, a vertical plane for instance. Move the octant in that plane (raising or lowering the sight) and you will be sure to find the moon's image somewhere. Having once found the reflected image of the moon, then turn the octant round the incident ray (continually changing the position of its plane) but so as still to keep sight of the moon in the horizon glass, and you will see, through the transparent part of it, all those stars which have the distance shown by the index. The star to be observed will lie in a line nearly perpendicular to the horns of the moon or her longer axis. As the star is always a bright one, and the circle in which it lies may be thus traced out, there will be no danger of mistaking it.

The directions before given (par. 139 and the following) hold exactly the same, whether you use plain sights or a telescope.

154. In these observations great care must be taken to keep the octant in the proper

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plane,

plane, according to par. 108. In common octants, where the back vane stands near the horizon glass, and has only a plain hole, it is not difficult to look so obliquely as to make a difference in the observed distance of half a degree, or more; and almost as great a difference may be made in the fore observation, even though a tube or telescope directs the sight. The positions in which the instrument must be held, to make these observations for the Longitude, are so various, and some of them so uncouth, that it may be many years before seamen in general can take these distances as exactly as they do now take an altitude of the sun. Nevertheless the great dexterity required in some arts daily exercised may convince us, that there is no manual operation in which the utmost accuracy may not at last be attained by constant practice. * In the Philosoph. Transactions, Nr. 425, the inventor of this instrument gives a rule for finding the error occasioned by holding it out of the proper plane. Every such rule requires a previous

* Nothing shows this more plainly than the astonishing degree of accuracy and perfection to which the art of playing upon musical instruments (of all sorts) is now carried.

knowledge of the deviation from that plane, which is not easy to get exactly, especially at sea. It is better to trust to the skill of the observer in holding it properly, and to such a construction of the instrument as will not permit an unskilful person to see the object at all, in those cases where the error would be very great.

Astronomical observations at LAND.

To observe the altitude of the sun by reflection from water.

155. The water must be sheltered from the action of the wind by the apparatus described in par. 33. The water-trough *p q r s*, Fig. 9. must be set upon the ground, or some firm wall, with its sides nearly in the sun's azimuth, which will be known by their casting no shadow. The box, Fig. 8, must then be set over it, but so as not to touch the trough. The glass planes must be set to the expected angle, and then put within the box so as to cover

the trough.* The eye must be defended with such a smoaked glass as is mentioned in par. 16, and one of the light-red glasses let down. By this means the image of the sun reflected from the water will appear yellow, and the image reflected from the mirrors will appear red: this is of great use to distinguish them from each other. The fore observation must be used if the sun's altitude be less than 45 degrees, but the back observation if that altitude be greater than 45 degrees.

156. Hold now the octant in a vertical plane, and direct the sight to the sun's image in the water; then move the index till you see the other image reflected by the mirrors, which will be known by its red colour. If there should be any difficulty in finding either image, put aside the smoaked glass in one case, and the red glass in the other, and you will be sure to see the sun. When you have found both images, then, if you use a telescope, make them as distinct as

* The sun's altitude at noon can always be guessed near enough to set the glass planes. At other times you may make a rough observation first, to find the altitude nearly, and then repeat it with the planes set exact. Four or five degrees in the angle of the planes is of no consequence.

you can; * likewise raise or fall it, till both images are equally bright. See par. 16.

157. All things thus prepared, move the index so as to bring the edges of the two images into contact, making the octant vibrate a little to distinguish that contact more perfectly, as before directed. Observe now whether the red sun or the yellow sun be uppermost: for the index will show double the altitude of the sun's upper limb, if the yellow image be uppermost; but double the altitude of the lower limb, if the red image be uppermost. This is on a supposition that you use plain sights; the contrary is the rule if you use a telescope that inverts.

158. Having thus found the double altitude, apply to it the correction for the index error by the rules in par. 54 and 65, and half of this gives the observed altitude of the sun's upper or lower limb, according as the yellow or red image was uppermost. Here remember to correct the double altitude for the index error before you halve it: if you will halve it first, then you must correct

* Both images cannot be perfectly distinct, because of the difference of their colour, but they will be very nearly so, if the telescope does not magnify too much.

that

that half by applying only *half* the index error found by the rules before mentioned.

159. To this altitude you must apply the correction for the sun's semidiameter, to be found in the Nautical Ephemeris at the third page belonging to every month. This correction is to be added if the altitude of the lower limb was observed, but subtracted if the upper limb was observed, and then you have the apparent altitude of the sun's center. This being further corrected, for refraction and parallax, by the rules referred to in par. 121 and 122, you have the **TRUE ALTITUDE** of the sun's center, from whence the Latitude of the place or hour of the day may be derived. *

160. If there be any suspicion that the glass planes are not ground true, repeat the observation, but change their places, so that the plane which before received the sun's direct rays, shall now receive the rays reflected from the water, and contrariwise; for the refraction of the planes will now have a contrary effect, and, if it before increased the observed altitude, will now di-

* See an example of determining the Latitude by this instrument in the Phil. Trans. Vol. LIX. page 239.

minish it. By thus changing the position of the planes, and arguing from a number of observations, either the Latitude of the place or the hour of the day may be got very near the truth.

161. If the octant be held in the hand only, it will be necessary to rest the arms on somewhat, and likewise to set the limb of the octant on a support; so that one hand may keep the octant in the sun's azimuth (holding it as directed in par. 107) while the other governs the index. But all this trouble may be avoided by mounting the octant on a stand, as before suggested; and then the observation may be made with ease and accuracy.

162. Some recommend treacle, or treacle diluted with water, to make the reflection. But if the fluid be so thick and tenacious that its surface cannot be disturbed by the wind, then there will be much uncertainty whether it be level; and if the liquor be so much diluted as to put this out of doubt, then the least breath of wind will raise tremors: so that glass planes, though expensive, are, I fear, necessary.

To observe the altitude of a star.

163. The process is exactly the same as before, only here you must use quicksilver, the reflection from water not being bright enough, unless for a few stars of the first magnitude. The quicksilver is very apt to collect dust on its surface: it should be strained through a silk handkerchief just before the observation.

164. The double altitude of the star must be first corrected for the index error by par. 54 or 65, and then halved. That half, being corrected for refraction as before, par. 121, gives the true altitude.

To observe corresponding altitudes of the sun.

165. This is one of the most useful operations in practical astronomy. You may by this means examine the going of a clock, and determine its error without a meridian line. The exact time of any appearance in the heavens is always a capital circumstance in astronomical observations. A method of determining that time, without the apparatus of a fixed observatory, is therefore very useful, especially to those who

go abroad. * Corresponding altitudes are likewise very useful in finding a meridian line, or adjusting the position of a transit telescope. These observations are not only useful but easy to make with almost any octant: for they require no rectification of the glasses, no accuracy in the divisions on the limb, nor any great truth in the planes that shade the water.

166. These altitudes should be observed (in our Latitude) at least two hours distant from noon, and therefore the fore horizon glass is always to be used. The best time is when the sun is on or near the *Prime Vertical*, that is, the East or West points of the compass.

167. Having prepared all things in the morning as before mentioned, par. 155, direct the sight (which we will here suppose to be the plain tube) to the sun's image in the water. Move the index till you see the image reflected by the mirrors, or the red sun. Bring down this red sun (by moving

* If a traveller takes with him this octant, a small telescope, and a portable time-piece (a spring clock for instance) to keep time for a day or two only, he may make very useful observations, without more expensive and cumbersome instruments.

the

the index) a little below the yellow sun, and fix the index by the setting screw.

168. Wait now till the upper limb of the red sun, by rising, just touches the lower limb of the yellow sun, making the red image now and then vibrate across the yellow one, to be sure of the contact of their nearest edges. Then note the hour, minute, and second, shown by the clock, either by calling to your assistant at the clock, at the instant you see the contact of the images, or by causing him to count the seconds aloud.

169. Again wait till the red sun, by rising, just covers the yellow sun, which may be easily known as the images are of two colours, especially if you now and then make the red image vibrate sidewise as before directed. Note the time by the clock as before.

170. Lastly wait till the lower edge of the red sun just touches the upper edge of the yellow sun; note the time by the clock, and you have finished one *set* of morning observations. Enter these down in one column under the title of *Eastern Azimuth*. Examine now the degrees and minutes shown by the index, and enter it at the
head

head of the whole, under the title of double altitude.

171. If you chuse to make more than one set of observations in that day, then release the index, and, setting it forward, repeat the former work, always remembering to *leave the index fixed* when the last set of morning observations is compleated.

172. Near the corresponding time in the afternoon (at which you may make a rough guess) prepare the water apparatus as before, par. 155, and be careful to set the glass planes, that cover the water, in the same position as before in the morning; that is, turn the same plane towards the sun as before. Take now the octant with the index yet fixed, and direct the sight to the sun's image in the water, and you will find the red sun uppermost, but may easily perceive it to sink so as to approach the yellow sun. Wait for the contact of the lower edge of the red sun with the upper edge of the yellow one, and note the time shown by the clock as before. In like manner note the time of the coincidence of the two images, also the time of the contact of the upper edge of the red sun with the lower

lower edge of the yellow one; and you have one compleat set (of the times) of corresponding altitudes of the sun.

173. Release now the index and set it carefully to the degree and minute at which it before stood during the set of morning observations preceding this last, and then you are prepared for the afternoon observations corresponding to that set, and which come on next: carry these on as before, and so till every set is compleated.

174. Enter the times of the afternoon observations in the same line of your paper with the morning observations to which they correspond, but in a different column, under the title of *Western Azimuth*. Thus when you begin in the morning enter the successive times on the left hand, one below another, descending as you proceed. In the afternoon begin on the right hand of these at the bottom line, ascending as you proceed; an example of which you may see in the Philosophical Transactions, Vol. LIX, page 237, and Vol. LX, page 357.

175. Take now the first pair of corresponding times and add them together; halve their sum, to which add six hours,
and

and it gives the time of solar noon derived from this pair; therefore set it down on the same line with this pair, but in another column under the title of *Meridian*. Do the same by the other two pairs in the first set. Take the mean of these three times of solar noon, correct it for the change of the sun's declination between the morning and afternoon observations, * and you have the time, shown by the clock at solar noon, inferred from the first set of observations. In the same manner may that time be inferred from each separate set. Take the mean of all the times thus found from each separate set, and you will have the exact time shown by the clock at solar noon, that is, when the center of the sun is upon the meridian. Look now in the Nautical Ephemeris for the equation of time, which you will find in the second page belonging to every month. Add or subtract this from twelve hours (according to the title at the head of the column in the Ephemeris) and

* A table, giving this correction for the Latitude of London and for every six degrees of the sun's Longitude, is in the *Astronomical observations*, &c. printed for T. Cadell. A table, from whence this correction may be easily computed for all Latitudes, is given in the Nautical Ephemeris for 1769.

you have the true time of solar noon, or the time that would be shown when the sun is upon the meridian, by a clock that went perfectly right. Compare this with the time actually shown at the same instant by your own clock (found as before) and you will have its error at the solar noon of that day with great exactness.

176. If the different sets of observations, made on the same day, follow each other within an hour at the Equinoxes, or within two near the solstices, you need not correct each set separately. It will be sufficient to take one mean of the whole and correct that as before.

177. If you use a telescope that inverts, the motions of the sun will be contrary to those here described. Thus the red sun will fall in the morning and rise in the afternoon.

178. Corresponding altitudes of the principal stars may be taken in like manner by reflection from quicksilver. You may in the same night take several observations of some one capital star at different altitudes, removing the index for each observation; or, which is better, keep the index always
fixed,

fixed, and take corresponding observations of different stars as they arrive successively at the same altitude. No correction for change of declination is wanted in this case. The time of a star's passage over the meridian will be precisely equidistant from the times of its attaining an equal altitude on the Eastern and Western Azimuths.

Of taking distances of the stars from each other.

179. Distances of the planets or fixed stars from each other may be easily taken by the octant, if mounted on such a stand as is described in the *Astronomical observations*, &c. After the octant is got into the proper position (by the directions given in that book) set the index to 0, and then direct the sight to the brighter of the two stars. Stir the index a little to get sight of the reflected image of that star. Continue to advance the index, and at the same time follow that image with your eye, turning the octant round the axis QQ (plate first of that book) till the reflected image is arrived at the other star, which you will see by direct vision. Make the two stars coincide, as

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directed par. 113, and the index will show on the limb their observed distance. This process resembles that more particularly described in par. 135.

180. Distances so observed may be very useful to determine the orbit of a comet. The motion of comets is such that they can seldom be seen on the meridian; and when they cannot this is the best way of determining their places in the heavens. It were to be wished that the use of this instrument in taking such distances was every where known. The stay of those extraordinary bodies in our regions is often so short, that, if the sky be unfavourable a few nights at our great observatories, we can learn nothing satisfactory about them; those who chance to see them at other places not knowing how to ascertain the circumstances upon which the investigation of their motions must be founded. These distances from the fixed stars must be corrected for index error and refraction. The observer must carefully note the former; astronomers will supply the latter, if either the altitude of the stars or the hour of the night be known. Those who would calculate the effect

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effect of refraction upon the distance, may consult the tables belonging to the Nautical Ephemeris, page 19.

Topographical observations, &c.

The use of the octant in county surveys.

181. The octant is of some use in taking the angles between distant objects on the horizon. Dark objects against a bright sky, or bright objects against a dark sky, are easily distinguished even by the naked eye, and the coincidence of their images in the octant may be well ascertained. But objects below the horizon can hardly be seen at all in the octant, especially if small or distant. The objects seen directly lose much of their light by its being transmitted through the upper part of the horizon glass, which had better be cut off, as was before observed, par. 13.

182. It is proper to take the distance of every object from two or three others, and likewise the distances of these last mutually.

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Thus take the distance of *A* from *B* and *C*, then the distance of *B* from *C* and *D* &c. Thus the observations will check each other. Not that the sum of the distances of several intermediate objects will always be equal to the distance of the extremes; or that the sum of the angles taken round the horizon will always make 360 degrees. This is not to be expected, be the observations ever so exact, unless the objects all stood in one plane. Yet the method before mentioned is a good check against gross errors, and particularly against mistakes in reading off the angles. These distances need no other correction but for index error. If indeed you could take the elevation or depression of every object, you might further correct the observations on that account, but it would greatly increase the work, and this correction for distant objects is so small that it may well be omitted.

183. Although the octant will not take angles between distant objects with all the accuracy that Astronomers require, yet it does the work of a common survey moderately well, and very expeditiously. And if a more exact survey was to be taken, with

with a better instrument, yet even then the octant would be of great use to make a rough survey first, in order to judge where the *principal stations* should be fixed. There ought to be five or six of these stations in every good survey, so chosen as to be every one in sight of all the rest, and such that you may command any part of the county from (at least) two of them. The places of these principal stations may be laid down by a larger instrument furnished with telescopic sights. The octant may then do very well for inserting villages and places of lesser note, whose distances will probably be near some or other of the principal stations.

184. It is often useful in these surveys to continue a right line quite through a county. To do this, we must determine where a line, drawn from a given point in the horizon through the eye of the surveyor, will again cut the horizon. This is no other than to set off an angle of 180 degrees from a given object. Set the index to 180 degrees (according to the back observation par. 24) and applying the eye to the back vane direct the sight to the given
 I 4 object.

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object. Note what point of the horizon, seen by reflection, coincides with the proposed object seen directly, for that is the point where the line again cuts the horizon. Otherwise you may direct the sight to the part of the horizon nearly opposite to the given object, and then observe what point seen directly coincides with that object now seen by reflection. One or other of these methods is preferable as the proposed object, or the part of the horizon opposite to it, is more or less bright. * By this means you may trace a meridian line thro' a county, or even through a kingdom, as has been done in France and Italy; and though this instrument will not determine that line very exactly, yet it is exceeding useful to find by this means what eminences it passes over, that you may know where to convey more expensive instruments, whose

* In strictness a line drawn from the object seen by reflection to the center of the index glass, is in this case parallel to a line drawn from the object seen directly through the center of the horizon glass, or through the eye of the observer. These two lines therefore do not make one and the same line continued: but as their perpendicular distance can never be more than five or six inches, in these surveys they may be accounted as one and the same line. An error of five or six inches, in the course of a line 15 or 20 miles long, is nothing.

size

size and delicacy does not admit of their being carried about without trouble and danger.

185. It is of great use to a surveyor to know when he is in the line joining two distant objects. In county surveys it is often difficult to know what a very distant object is, especially if it be seen only just above some intermediate eminence. The only sure way to ascertain a place so seen is to go to that intermediate eminence, from whence you may view that place at a much nearer distance, and in the direct line: for unless you see it in the same direction, the appearance will be so much changed that it may not be known. Here then a method of finding when you are exactly in that line is of great use. When therefore you are near the intermediate place, set the index of the octant to 180 degrees as before; direct the sight to any object you suppose may be that sought, and observe whether it now coincides with your former station seen by reflection: if not, change your place till it does, and then you are in the line joining your former station and the object. Otherwise you may direct the sight to your former station and observe the distant object
by

by reflection, just as one or the other is more easily discerned, *

The use of the octant in surveying land.

186. These surveys are sometimes carried on by taking angles, sometimes by measurement only. The best surveyors use both methods as a check upon each other. The octant will take these angles more expeditiously and with far less trouble than a

* The octant will do this business very well, but I found it useful to have a much lighter and smaller instrument made for this purpose only: one that may be carried in the pocket, and used on horseback. Mine (which may be called a *back-liner*) is a square tube, $1\frac{1}{2}$ inches in the side, $8\frac{1}{2}$ inches long, and weighs half a pound.

In taking angles from *Bardon* hill in *Leicestershire* I saw some very distant land, just peeping over (*Cannock*) *Cank beath* in *Staffordshire*. *Cank beath* is 25 miles distant from *Bardon*: the land over it could be seen only by the light of the setting sun almost behind it, nor then but by the assistance of a good telescope that magnified 20 times. Here then this instrument was of great use to find the part of *Cank beath* over which the line passed that joined the station at *Bardon* and the object seen over the heath. In seeking this place it was easy to see on horseback when you approached and when you passed beyond this line. The place sought, over which the line passed, proved to be a part of *Cank beath* near *Style-Copse*, about half way between *Cank* and *Litchfield*. From this place the object could be plainly seen. It put on the same appearance, and was well known in that country to be the *Wreken* in *Shropshire*.

Bardon is by estimation 50 miles distant from the *Wreken*, which I believe is as far as any two objects can be seen from each other in England.

At *Bardon*, the *Wreken* lies to the right of *Litchfield*; the angle between them is three degrees, as near as it could be guessed from the field of the telescope.

Theodolite, because that instrument requires steadiness: but then these angles will be taken in the several planes in which the objects lie, and not reduced to an horizontal plane as in the Theodolite. If the ground be very uneven, this may occasion errors in plotting the lordship, and consequently in the measurement taken from such a plot. The several elevations and depressions might indeed be taken, and the angles reduced by calculation to the horizontal plane, but this correction greatly increases the work.

187. Besides the forementioned corrections, another may be necessary if the objects are very near. For the octant shows on the limb the angle which a ray, drawn from the reflected object through the center of the index glass, makes with a ray drawn from the other object through the center of the horizon glass, or, which is the same, through the sight-vane. * Now the eye is always placed at the sight-vane: therefore when the former of those lines intersects the latter just at the sight-vane, then the octant shows the exact angle which the

* See the note at the end of par. 93, Nr 1.

objects

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objects would subtend to the eye so placed, were the instrument removed. If the intersection of those lines falls elsewhere, there will be a difference between the angle shown on the limb and that which the objects subtend to the naked eye. This difference is seldom worth considering, however the rule for finding the correction necessary on that account is in the *Astronomical observations*, &c. par. 105. That rule relates to fore observations only; a similar correction should be made in back observations, the rule for which may be easily derived from what is there laid down.

188. When a survey is made by measurement only, it is usual to mark out a right line near every boundary fence; to this line perpendiculars are let fall from every bend in the fence. Such a line is called a *station line*, each end of it a *station*, at which is placed a *station-staff*. The perpendiculars are called *off-sets*, and are measured with an *off-set staff*. The distances of the several points where the off-sets cut the station line (reckoned from the first station) are all measured with the chain. A line being then drawn upon paper to represent the

station line, the several perpendiculars are set off at their proper distances by a *plotting-scale*. Join the extremities of all these perpendiculars, and you will have a plan of the boundary similar to the real one. And thus a whole Lordship may be plotted.

189. In these surveys it is useful to run a line through the middle of the Lordship, and perhaps another across it, and then to take off-sets from those lines to any remarkable points, especially in the fence that bounds the whole. This keeps the map of the whole in its just form. For though the several enclosures may be separately laid down near enough to ascertain the quantity of acres they contain, yet, if small errors be committed in the form of each, when the whole is set together, error will be heaped upon error, and the whole will by no means resemble the true shape of the Lordship.

190. So many off-sets being necessary, a method of taking them readily is valuable. Nothing can do it more expeditiously than this octant. Adjust the instrument and set the index to 90 degrees; let the surveyor walk along the station-line with the octant
in

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in his hand, always directing the sight to the further station-staff: let his assistant walk along the boundary-line. Then, if the surveyor would make an off-set from a given point in the station-line, let him stop at that place and wait till he sees his assistant by reflection in the octant, and the assistant is then at the point in the boundary through which that off-set passes. On the other hand, if the surveyor would take an off-set from a given point or bend in the boundary, let the assistant stop at that place, and let the surveyor walk on in the station-line till he sees his assistant by reflection in the octant, and that will be the point where an off-set from the proposed point or bend will fall.

191. If you would have these points determined with great precision, then upon the plane of the octant draw a line from the sight-vane through the middle of the horizon glass. From the center of the index glass draw another line, cutting the former at right angles. Mark their intersection and drill a small hole through it. *

* A small bar should be left in the pattern by the maker, for the sake of this hole.

From

From this hole suspend a pointed plummet; bring this over the station-line, and it will determine the place of the off-set to a quarter of an inch. The octant in this case should be placed on the stand described par. 31; the plumb-line will fall clear of the head of the stand. If you would make an off-set from a given point in the station-line, set up a spiked staff in that point; place this hole in the octant on the top of the spike; then, directing the sight along the station-line, you will see by reflection the point in the boundary-line where the off-set cuts it. In some circumstances it may be necessary to turn the face of the octant downwards, but this is no inconvenience if you use a stand. By this means you may make off-sets (that is, erect perpendiculars to a given line) in a garden, or even in an house, with sufficient exactness for the purposes of building.

192. These off-set lines are drawn, as we said, at right angles to the station-line. But in some cases it may be necessary to draw a line to answer the purpose of an off-set, which shall make some other given angle with the station-line. In this case
you

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you have nothing to do but to set the index to the proposed angle, and proceed as before. The line on the surface of the octant, passing through the center of the index glass, must now be drawn so as to intersect the other line (passing through the sight-vane and horizon glass) in the given angle; and then that intersection will determine the angular point through which the hole must be drilled to ascertain the place of the offset as before.

To take the height of a building by reflection from water.

193. The angle of the elevation of the building may be taken by the octant in a way much resembling that described in par. 155. The water-trough must stand on the ground, and the box be set over it so as not to touch the trough. The glass planes must be set to the expected angle, and put within the box so as to cover the trough. Hold now the octant in a vertical plane, direct the sight to the image in the water, and move the index till you see the other image reflected by the mirrors. These images will be known from each other by their position; the image reflected by
the

the water will be inverted, that by the mirrors erect, if you use plain sights; the contrary if you use a telescope that inverts.

194. Having found both images, bring their extremities into contact, making the octant vibrate as before to ascertain this contact; the index will then show the double elevation of the building above the surface of the water, or double the angle it makes with the horizon supposing the eye placed at the point of reflection from the water. This angle must be corrected for the index error before you halve it.

195. The octant should be held as near the water as may be to lessen another error we are going to consider. For if the object be so near that the two points of reflection (that from the water, and that from the index glass) cannot be considered as one, a further correction will be necessary. Estimate the distance of the top of the building from the water, by an observation taken as before. From this and the distance between the water and index glass at the time of observation, you must compute the angle which the two points of reflection subtend at the top of the building. It is not diffi-

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cult to see exactly from what part of the water the reflection is made; but we will suppose that reflection to be always made from the middle of the water-trough, and the other from the middle of the index glass. Make then an estimate of the angle which the middle of the water-trough and the middle of the index glass subtend at the top of the building when the octant is in the position in which the observation was made. Add this angle to that shown on the limb (whether by the fore or back observation *); correct the whole for index error and then halve it. This half is the exact angle which a ray, drawn from the top of the building to the point of reflection on the water, makes with the surface of the water; or it is the angle of the elevation of the building above the level of the water, supposing the eye placed at the point of reflection or middle of the water-trough. If the distance of this point from the bottom of the building be known, the height of the building

* In the back observation it is possible for the ray, incident on the water, to pass between the index glass and the eye: shading this interval by the hand will in that case hide the image in the water, and show whether it be so or not. This case is not probable; but if it should happen, then this angle is to be *subtracted* from that shown on the limb.

above the level of the water is easily found by plane Trigonometry.

P O S T S C R I P T.

Of the errors which arise from not adjusting the index glass.

196. In par. 44 we have shown the method of setting the index glass perpendicular to the plane of the octant. As some instruments have no provision for doing this, the reader may wish to be informed what errors will arise from neglecting this adjustment. Now if the two adjustments of the horizon glass, described par. 48 and 51, be properly made, then the glasses will be parallel to each other when the index stands at the beginning of the divisions; but both of them may deviate (equally) from the perpendicular to the plane of the octant.

197. In this case the index will not show on the limb the double inclination of the speculums. For their common section is never perpendicular to the plane of the octant, nor to both the sections of the reflecting planes made by the plane of the

octant. Therefore the angle between these last mentioned sections, equal to half the angle shown by the index, is not that measure of the inclination of the reflecting planes to each other, which is laid down by *Euclid* in the sixth definition of the eleventh book of the *Elements*; nor does the index show, as it ought, the double of their inclination. To correct this, say, as radius is to the cosine of deviation, so is the sine of one fourth of the angle shown on the limb, to the sine of one fourth of the corrected angle; which is therefore always less than that shown on the limb.

198. But besides this, another and greater error will arise if you look through a tube or telescope whose axis is parallel to the plane of the octant. All observations should be made in a plane perpendicular to both the speculums, and then the objects may be considered (according to Hadley's method, *Phil. Trans.* Nr 420 and 425) as lying in the circumference of a great circle in the heavens, whose plane is perpendicular to the common section of the speculums. For this end, the telescope should be in that plane; but in the case we have supposed,
the

the telescope is inclined to that plane: Therefore the objects must here be considered (in Hadley's way) as lying in a parallel circle, whose distance from the great circle is the angle which the telescope makes with that plane; or, which is the same thing, the complement of that distance is equal to the angle which a line, drawn through the common section of the speculums, parallel to the axis of the telescope, makes with that common section.

199. To find this angle, besides the angle of the deviation of the glasses and the angle shown by the index, we must likewise know the angle which a line, drawn through the common section of the plane of the octant and horizon glass, parallel to the axis of the telescope, makes with that section. This will be nearly the complement of the angle of incidence on the horizon glass, and may be supposed equal to it. From these data we may compute the distance of the parallel from the great circle before mentioned, and from thence (by Hadley's fifth corollary) the true angle between the objects.

200. An example may explain this. Let the

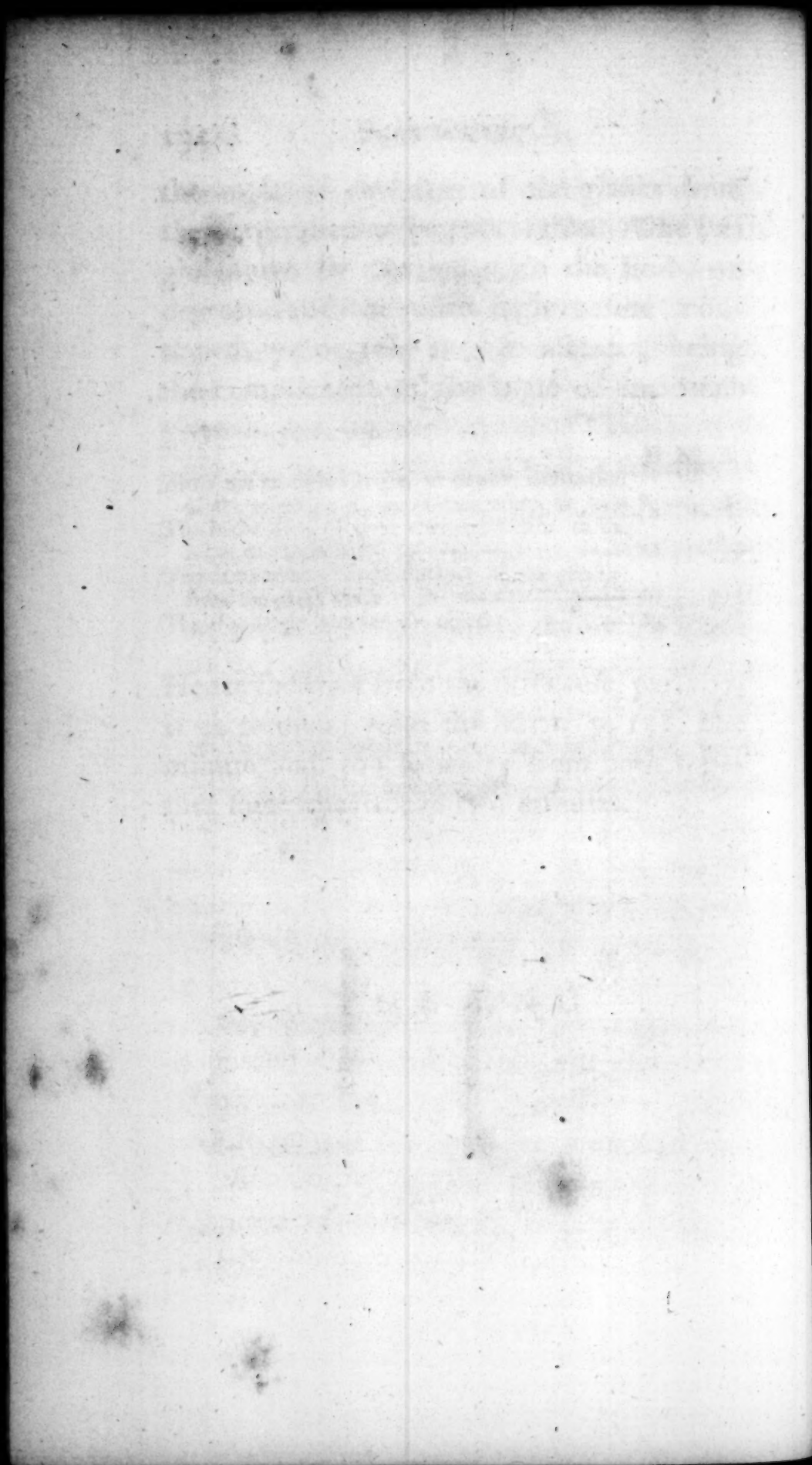
the angle of deviation of the glasses from the perpendicular be one degree. The angle shown by the index on the limb, 90 degrees; and the third angle before mentioned, 74 degrees and 30 minutes, being the complement of the angle of incidence given in par. 95.

	D	M	S
Then the corrected angle, or double inclination of the reflecting planes to each other, is	—	89	59 2
The inclination of their common section to the plane of the octant	—	88	55 3½
The complement of the distance of the parallel from the great circle	—	88	55 32½
The true angle between the objects	—	89	57 55½

Hence the error from the first cause, par. 197, is 52 seconds; from the latter, p. 198, one minute and 12½ seconds; from both together somewhat above two minutes.

T H E E N D





ADDITIONS TO THE DIRECTIONS,
&c.

To come in after par. 187.

In adjusting the fore horizon glass, if instead of a distant object you adjust it by that object which is to be viewed directly, following in other respects the directions in par. 52; the octant will then show, on the limb, the exact angle which the objects subtend at the center of the index-glass; and that whether they are near or distant. This indeed is not the angle which those objects would subtend to the naked eye placed at the sight vane; but it is sometimes useful to determine the angle which they subtend at the center of the index glass.

Otherwise, if you have no mind to alter the adjustment already made by a distant object, then find the index error as directed in par. 54; only using, for this purpose, that object which is to be viewed directly, instead of a distant one. The index error, so found, must be added to, or subtracted from,

from, the angle shown on the limb by the rule in par. 54. This will give the exact angle which the two objects subtend at the center of the index-glass as before.

The difference between the angle shown on the limb (supposing the horizon-glass adjusted by a distant object) and the angle which the objects subtend at the center of the index-glass, depends wholly on the distance of the object seen by direct vision. The difference between the former angle and that which the objects subtend to the naked eye (at the sight-vane) depends on the place of the object seen by reflexion, that is on its distance and the angle shown on the limb; not at all on the distance of the object seen directly.


To come in after par. 192.

If you find the index-error as directed in par. 54; only using, for this purpose, that object which is to be viewed directly instead of a distant one, and set the index so that the angle shown on the limb, when corrected for index-error, may be the proposed
angle

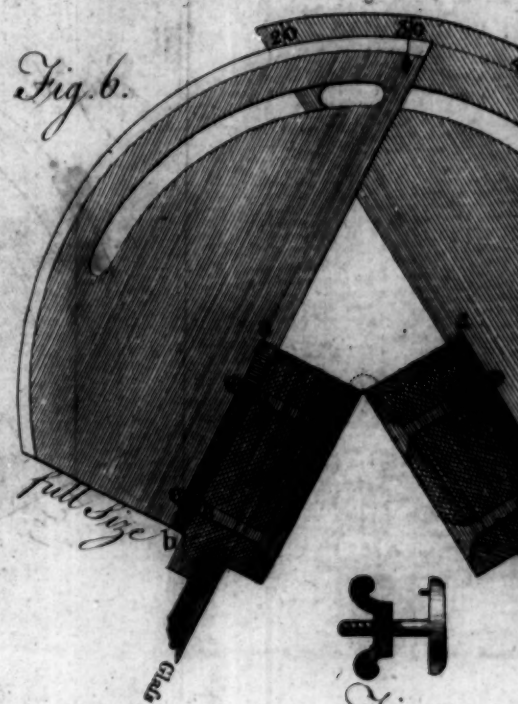
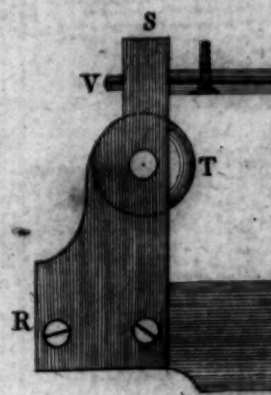
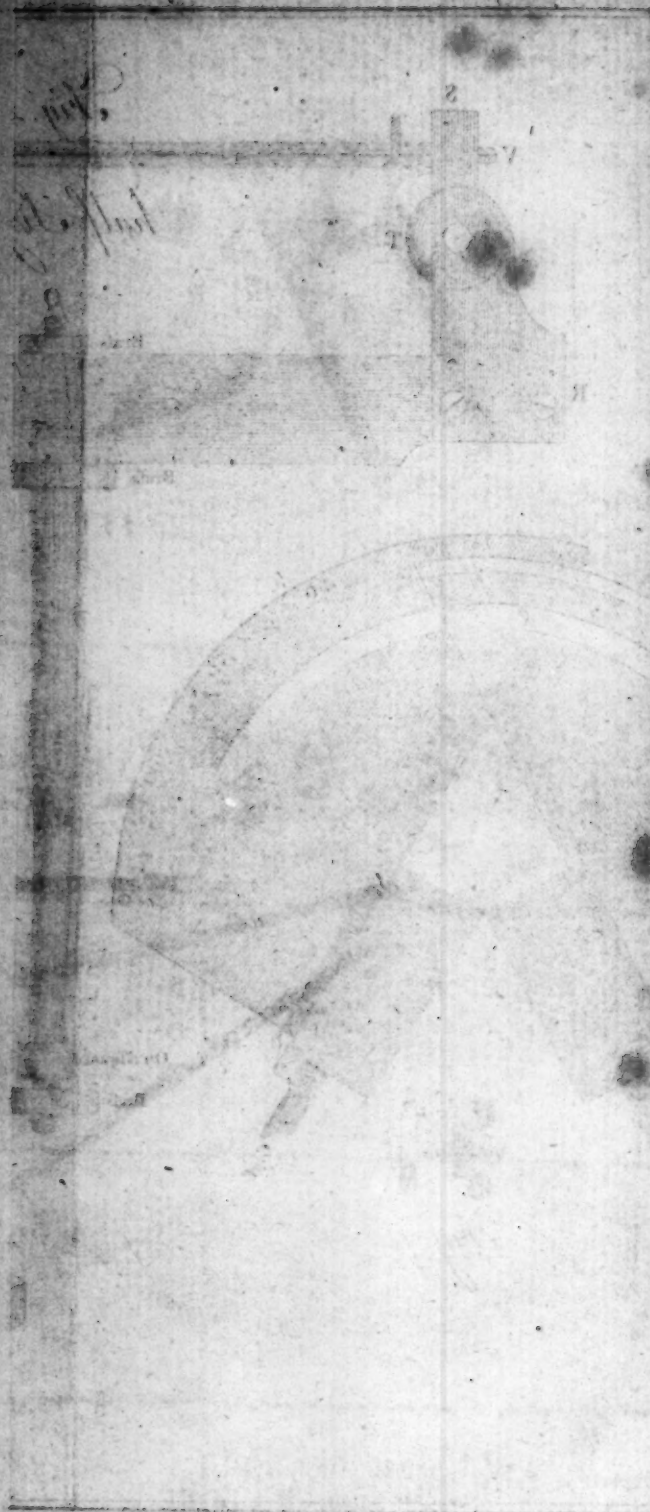
angle of the off-set; the angular point will then always fall on the center of the index-glass. By this means that point will be invariable whatever be the angle of the off-set; but then the index-error must be determined over again for every particular off-set that is to be made. Upon the whole, the method before described seems preferable.

*Read the note, Nr 2, at the end of par. 93,
as follows:*

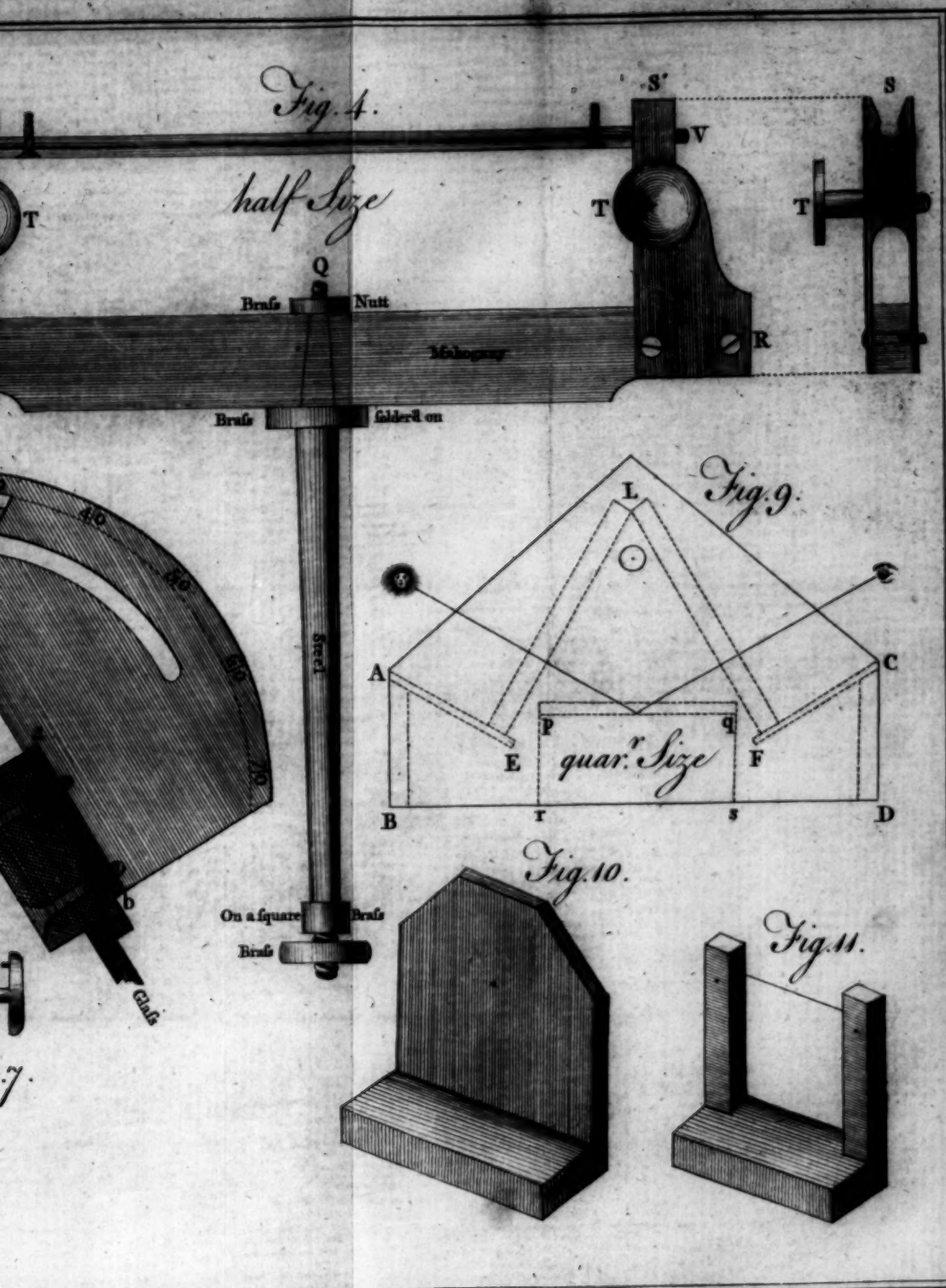
2. The same is true in the back observation, understanding by the inclination of the reflecting planes the *acute* angle they make with each other, according to *Euclid*, Def. 6. El. xi. Otherwise we may say, the obtuse angle between the incident ray and that ray after two reflections, is double the supplement (to 180 degrees) of the obtuse angle between the faces of the speculums. And this is the angle shown on the limb, if it be read off by the rule in par. 24.



1890. May 10. No. 10. 1890.



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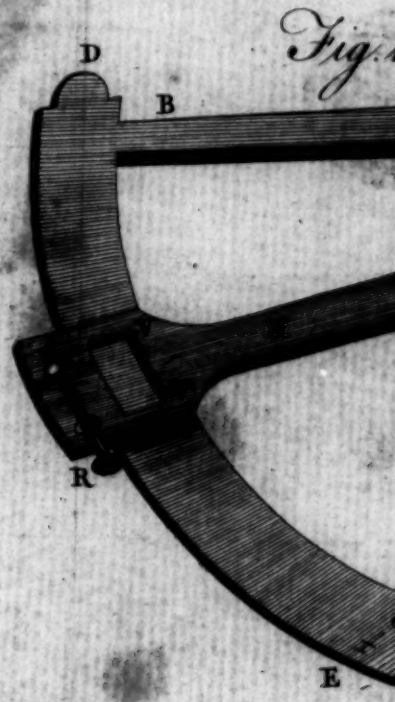
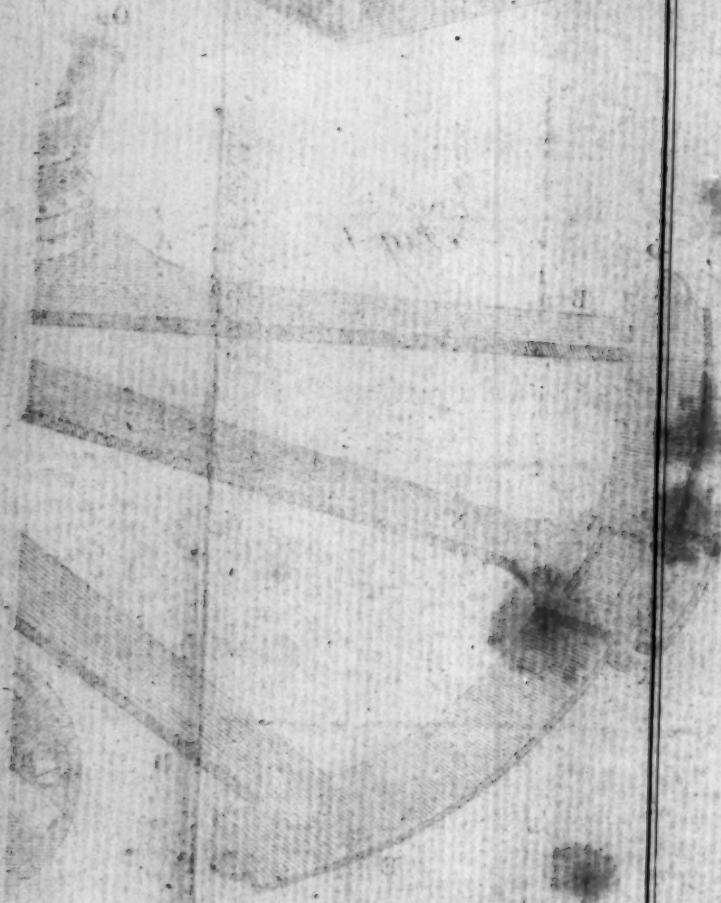
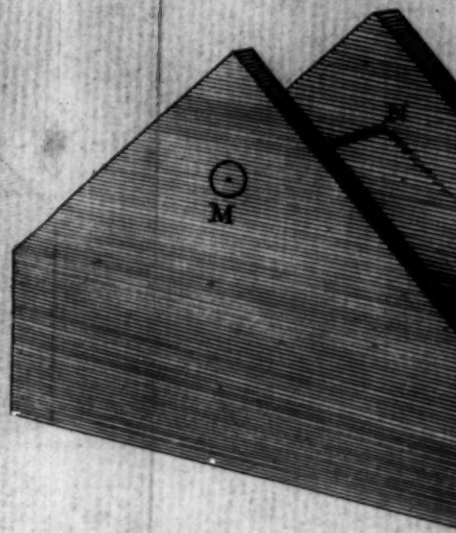
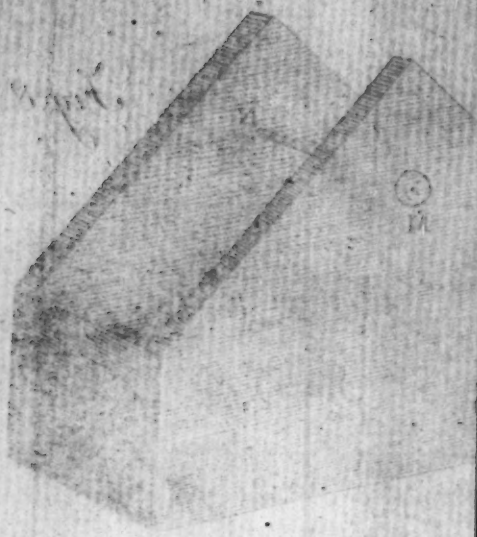
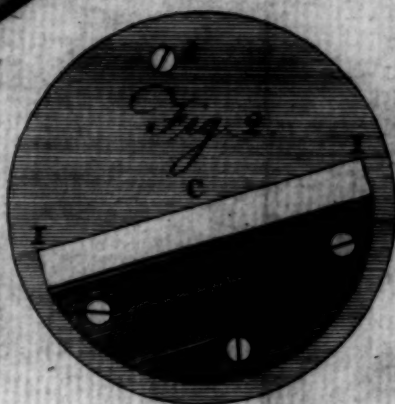
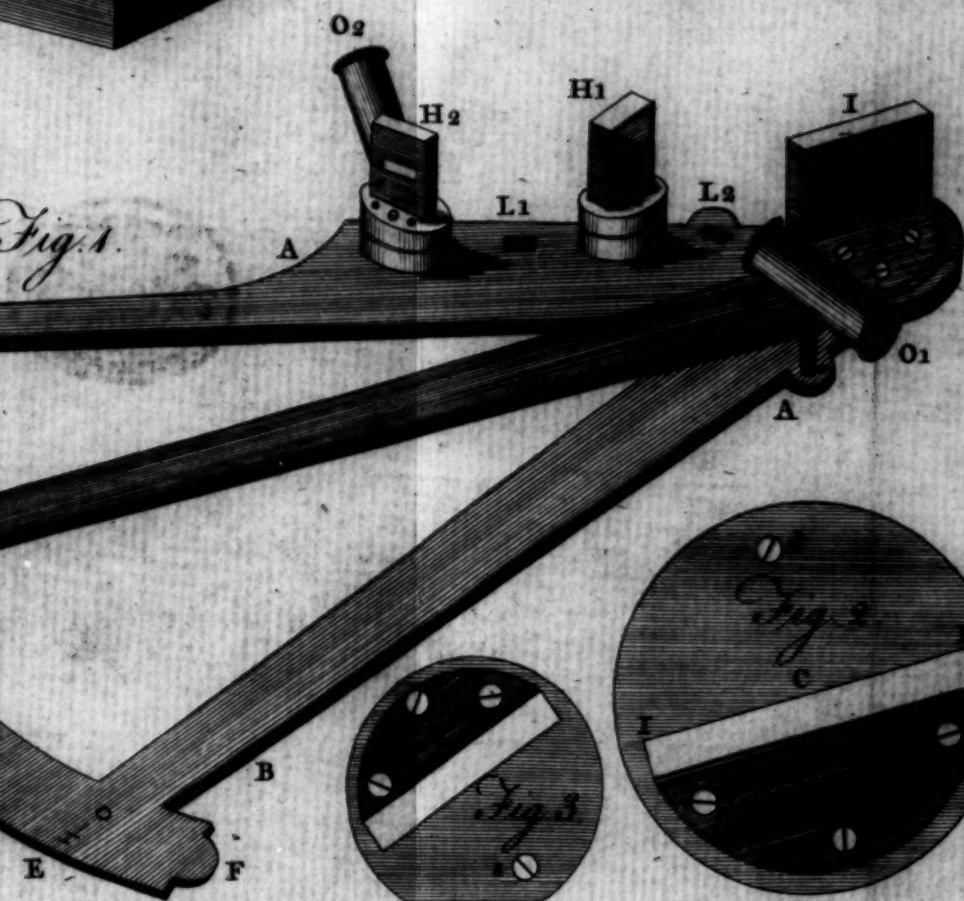
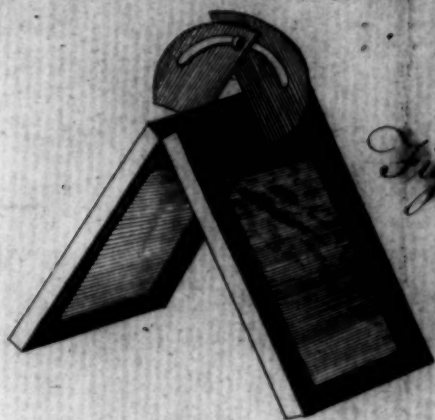


Fig.

Drawn by W.L.



Engraved by Whitechurch Bartholomew London.